CALCULATION OF THE STATE OF STRESS IN A WELDED ROTOR OF A STEAM TURBINE

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Methods have been described [1, 2] for calculating the state of stress in a welded rotor, which is considered as consisting of individual disks linked by thin-walled cylindrical shells. The transverse forces and bending moments at the edges of the shell are determined from the conditions for continuity in the radial displacement of the disk with the adjoining shell at the radius of the median surface, together with zero value for the rotation of the end sections of the shell.

Another method [3] involves considering the rotor as a thick-walled cylinder of constant cross section loaded at the points of attachment of the disks by radially distributed forces of constant magnitude.

These methods represent in a simple mathematical form the general state of stress in a welded rotor, but they do not take into account the actual possibility of bending in the disks.

Here we use the data of [2] to present a method of calculating the elastic state of stress in a welded rotor taking into account the bending of the disk and the finite thickness of the connecting parts. A relatively simple solution is obtained by approximating the actual disk shape by a power-law one.

We consider a disk as a uniformly heated isotropic plate of rotation having a variable thickness, and (1.93) of [4] gives

\[
\frac{d}{dr} \left( \frac{E r}{1 - \mu^2} \frac{du}{dr} + \frac{d}{dr} \left( \frac{\mu E h}{1 - \mu^2} \right) \frac{E h}{(1 - \mu^2)^2} \right) u = -\rho \omega^2 r^2 h,
\]

where \( u \) and \( \dot{h} \) are, respectively, the radial displacement and thickness of the disk at radius \( \dot{r} \); \( E = \text{const} \); \( \mu = \text{const} \); \( \sigma \) are the parameters of the elasticity and the density of the disk material; and \( \omega \) is the angular velocity of rotation.

We put \( u = u/R \) (\( R \) is the maximum radius of the disk); \( r = \dot{r}/R \), \( h = \dot{h}/R \), to get from this equation that

\[
\frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr} - \frac{1}{r^2} u = -\frac{d}{dr} (\ln h) \frac{du}{dr} - \frac{\mu}{r} \left( \frac{d}{dr} (\ln h) \frac{du}{dr} - \frac{1 - \mu^2}{E} \rho \omega^2 R^2. \right)
\]

The disk profile is approximated by

\[
h = h_0 e^{-\alpha r^2},
\]

where \( h_0 \) is the thickness at the initial radius; \( 0 < \alpha < 1 \) is a parameter; and \( e \) is a natural number.

We derive an approximate solution to (1) with (2) by the Foubini–Liouville–Steklov method by transformation to an integral Volterra equation of the second kind, using successive approximation [5, 6]:

\[
u = \gamma_1 \left[ -\frac{C^2}{2} \left( 1 - \mu \right) r^2 - \frac{\alpha^2}{36} \left( 1 + \mu \right) (5 + \mu) r^4 + \frac{\alpha^2}{36} \left( 1 - \mu \right) (5 + \mu) (9 + \mu) r^6 \right]
\]

\[
+ \gamma_2 \left[ \frac{1}{r} - \frac{\alpha^2}{240} \left( 1 - \mu \right) r^2 - \frac{\alpha^2}{36} \left( 1 - \mu \right) (3 + \mu) r^4 - \frac{\alpha^2}{36} \left( 1 - \mu \right) (3 + \mu) (7 + \mu) r^6 \right] - \frac{1 - \mu^2}{8E} \rho \omega^2 R^2,
\]

where \( \gamma_1 \) and \( \gamma_2 \) are constants to be determined from the boundary conditions. The maximum error is 0.2\% when this formula is applied to the disk of a welded rotor with \( \alpha = 0.3 \) and \( r = 1.0 \).

The radial and circumferential stresses are found from the following formulas:

\[
\sigma_r = \frac{E}{1 - \mu^2} \left( \frac{du}{dr} + \mu \frac{d\sigma}{r} \right) = \gamma_1 \frac{E}{1 - \mu} \left[ 1 + \frac{\alpha}{3} (5 + \mu) r^4 + \frac{\alpha^2}{4} (5 + \mu) (9 + \mu) r^6 \right. \\
+ \frac{\alpha^2}{7} (5 + \mu) (9 + \mu) (13 + \mu) r^{10} \left. + \gamma_2 \frac{E}{1 + \mu} \left[ \frac{1}{r} - \frac{1}{5} \right] - \frac{3 + \mu}{8} \rho \omega^2 \sigma^2 \right]
\]

(4)

\[
\sigma_\theta = \frac{E}{1 - \mu^2} \left( \frac{u}{r} + \mu \frac{du}{dr} \right) = \gamma_1 \frac{E}{1 - \mu} \left[ 1 + \frac{\alpha}{3} (1 + 5\mu) r^4 + \frac{\alpha^2}{4} (5 + \mu) (1 + 9\mu) r^6 \right. \\
+ \frac{\alpha^2}{7} (5 + \mu) (9 + \mu) (1 + 13\mu) r^{10} \left. + \gamma_2 \frac{E}{1 + \mu} \left[ \frac{1}{r} - \frac{1}{3} \right] - \frac{3 + \mu}{8} \rho \omega^2 \sigma^2 \right]
\]

(5)

Formulas (3)-(5) are shown by (2) to be suitable for a disk of constant thickness for \( \alpha = 0 \).

Figure 1 shows the general case of loading for a rotating disk, where \( N_{Ra} \) and \( N_{Rb} \) are the radial loads distributed over the circles \( 2\pi a \) and \( 2\pi b \).

From (4) with known \( \sigma_{Ra} = N_{Ra}/h_a R \) and \( \sigma_{Rb} = N_{Rb}/h_b R \) we get values for \( \gamma_1 \) and \( \gamma_2 \):

\[
\gamma_1 = \frac{1 - \mu \frac{E}{h_a R}}{1 - \mu} \cdot \left( \frac{N_{Ra} \frac{3 + \mu}{8} \rho \omega^2 \sigma^2 R^2}{A_1 B_3 - A_2 B_1} - \frac{N_{Rb} \frac{3 + \mu}{8} \rho \omega^2 \sigma^2 R^2}{A_1 B_4} \right)
\]

(6)

\[
\gamma_2 = \frac{1 - \mu \frac{E}{h_b R}}{1 - \mu} \cdot \left( \frac{N_{Rb} \frac{3 + \mu}{8} \rho \omega^2 \sigma^2 R^2}{A_1 B_3} - \frac{N_{Ra} \frac{3 + \mu}{8} \rho \omega^2 \sigma^2 R^2}{A_1 B_4} \right)
\]

(7)

where

\[
A_1 = 1 + \frac{\alpha}{3} (5 + \mu) a^4 + \frac{\alpha^2}{7} (5 + \mu) (9 + \mu) a^8 + \frac{\alpha^2}{1} (5 + \mu) (9 + \mu) (13 + \mu) a^{12};
\]

\[
A_2 = -\frac{1}{\mu^2} \frac{\alpha}{2} (3 + \mu) a^2 - \frac{\alpha^2}{4} (3 + \mu) (7 + \mu) a^6 - \frac{\alpha^2}{6} (3 + \mu) (7 + \mu) (11 + \mu) a^{10};
\]

\[
B_1 = 1 + \frac{\alpha}{3} (5 + \mu) b^4 + \frac{\alpha^2}{7} (5 + \mu) (9 + \mu) b^8 + \frac{\alpha^2}{1} (5 + \mu) (9 + \mu) (13 + \mu) b^{12};
\]

\[
B_2 = -\frac{1}{\mu^2} \frac{\alpha}{2} (3 + \mu) b^2 - \frac{\alpha^2}{4} (3 + \mu) (7 + \mu) b^6 - \frac{\alpha^2}{6} (3 + \mu) (7 + \mu) (11 + \mu) b^{10};
\]

To solve the boundary-value problem for a welded rotor [2], we derive from (3), (6), and (7) linear equations for \( u_a = f_1(N_{Ra}, N_{Rb}) \), \( u_b = f_2(N_{Ra}, N_{Rb}) \):

\[
u_a = \left( \frac{N_{Ra} \frac{3 + \mu}{8} \rho \omega^2 \sigma^2 R^2}{A_1 B_3} - \frac{(1 + \mu) C_1 B_2 - (1 + \mu) C_1 B_3}{E (A_1 B_3 - A_1 B_1)} \right)
\]

\[
\times \left( 1 - \mu \frac{E}{h_a R} \right) \frac{C_1 B_2 - (1 + \mu) C_1 B_3}{E (A_1 B_3 - A_1 B_1)} - \frac{1 - \mu^2}{8E} \rho \omega^2 \sigma^2 R^2,
\]

(8)

\[
u_b = \left( \frac{N_{Ra} \frac{3 + \mu}{8} \rho \omega^2 \sigma^2 R^2}{A_1 B_3} - \frac{(1 + \mu) C_1 B_2 - (1 + \mu) C_1 B_3}{E (A_1 B_3 - A_1 B_1)} \right)
\]

\[
\times \left( 1 - \mu \frac{E}{h_b R} \right) \frac{C_1 B_2 - (1 + \mu) C_1 B_3}{E (A_1 B_3 - A_1 B_1)} - \frac{1 - \mu^2}{8E} \rho \omega^2 \sigma^2 R^2,
\]

(9)

where

\[
C_1 = a + \frac{\alpha}{3} (1 + \mu) a^4 + \frac{\alpha^2}{7} (1 + \mu) (5 + \mu) a^8 + \frac{\alpha^2}{1} (1 + \mu) (5 + \mu) (9 + \mu) a^{12};
\]