Automated theorem proving in mathematics

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Abstract

The MUSCADET theorem prover is a knowledge-based system able to prove theorems in some non-trivial mathematical domains. The knowledge bases contain some general deduction strategies based on natural deduction, mathematical knowledge and metaknowledge. Metarules build new rules, easily usable by the inference engine, from formal definitions. Mathematical knowledge may be general or specific to some particular field. MUSCADET proved many theorems in set theory, mappings, relations, topology, geometry, and topological linear spaces. Some of the theorems were rather difficult. MUSCADET is now intended to become an assistant for mathematicians in discrete geometry for cellular automata. In order to evaluate the difficulty of such a work, researchers were observed while proving some lemmas, and MUSCADET was tested on easy ones. New methods have to be added to the knowledge base, such as reasoning by induction, but also new heuristics for splitting and reasoning by cases. It is also necessary to find good representations for some mathematical objects.

1. Introduction

Artificial intelligence and mathematics may be related in many ways. One of them is getting machines to do mathematics, and especially mathematical reasoning, for example theorem proving. The first provers, at the beginning of artificial intelligence, were tested on mathematical theorems. Then theorem proving was often studied from a theoretical point of view, by researchers in AI and also by logicians. Theories were built and their properties studied, in particular their complexity and theoretical efficiency. On the other hand, one may be interested rather in effectively proving theorems in some fields of mathematics, or more generally in automatizing mathematical reasoning. General proving methods are not sufficient, but methods of problem solving have to be used, especially all that is related to knowledge (knowledge representation, knowledge-based systems, expert systems). The MUSCADET system, written and experimented by the author, is a knowledge-based system which may receive mathematical knowledge and know-how in a convenient and modular manner. It was conceived to be able to prove difficult theorems in mathematics. It proved many theorems in set theory, mappings, relations, topology geometry, topological
linear spaces, and some of them were difficult. It is now being experimented in order that it may become an assistant for mathematicians in discrete geometry for cellular automata.

After briefly describing the MUSCADET system (section 2) and some of the theorems it proved (section 3), we present the domain in which mathematicians ask for an assistant (section 4) and how MUSCADET has to be improved for this new task (section 5).

2. The MUSCADET system

MUSCADET is described in detail in [13, 15]. Here, we shall describe its main features and some of the results obtained.

2.1. NATURAL DEDUCTION

Two families of methods may be used in theorem proving. The most known are methods based on the resolution principle [20]. The advantages of the resolution principle are its simplicity (there is only one inference rule) and its theoretical possibilities (it is complete for refutation in first-order predicate calculus). However, in practice it is often difficult to obtain proofs because of combinatorial explosion (number of generated clauses). A great deal of research has been done to improve resolution strategies, but other methods, based on natural deduction [3], are more adapted to mathematics. They are usually not equivalent to the rules of natural deduction of Gentzen, which is itself complete, but contain heuristic selections of them and possibly other rules. These methods look like those used by humans, so it is easier to fine good heuristics, to understand what is wrong in case of failure, and to communicate with the mathematicians. The notions of hypothesis and conclusion to be proved are privileged. A complicated theorem may be split into several easier ones. One method consists of deducing all that is possible until the conclusion is proved, and to transform the statements where necessary.

Here are some of the general deduction strategies of MUSCADET.

To prove $\forall x P(x)$, prove $P(x)$ for any $x$.

To prove $A \Rightarrow B$, assume $A$ and prove $B$.

To prove $A_1 \land \ldots \land A_n$, prove successively every $A_i$.

If the conclusion $C$ is to be proved, and $C$ is deduced from actual hypotheses, then the theorem is proved.

2.2. KNOWLEDGE BASES – METAKNOWLEDGE

MUSCADET was conceived as a knowledge-based system. It is composed of an inference engine and of several structured knowledge bases which express mathematical knowledge by means of rules and metarules.