DEFINITE CLAUSE PROGRAMS are CANONICAL *
(OVER A SUITABLE DOMAIN)

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Abstract

For each first-order language $L$ with a nonempty Herbrand universe, we construct an
algebra $\mathcal{C}$ interpreting the function symbols of $L$ that is a model of the Clark equality theory
with language $L$ and is canonical in the sense that for every definite clause program $P$ in the
language $L$, $T^\mathcal{C}_P \downarrow \omega$ is the greatest fixed point of $T^\mathcal{C}_P$. The universe of individuals in $\mathcal{C}$ is a
quotient of the set of terms of $L$ and is, a fortiori, countable if $L$ is countable. If $L$ contains
at least one function symbol of arity at least 2, then the graphs of partial recursive functions
on $\mathcal{C}$, suitably defined, are representable in a natural way as individuals in $\mathcal{C}$.

Keywords: Clause programs, partial recursive functions, semantics.

1. Introduction

There is an asymmetry in the model-theoretic semantics of completed definite
clause logic programs (cf. [8]). For those definite clause programs ** $P$ for which
$\Gamma_P \downarrow \omega$ is unequal to the greatest fixed point $\text{gfp}(T_P)$ of $T_P$ we have, for each
ground atomic formula $A$ in the language of $P$, that

$$zomp(P) \models \neg A \iff A \notin T_P \downarrow \omega$$

but only

$$A \notin T_P \downarrow \omega \Rightarrow A \notin \text{gfp}(T_P)$$

without the converse, while $P \models A$ iff $\text{comp}(P) \models A$ and

$$\text{comp}(P) \models A \iff A \in T_P \uparrow \omega,$$

$$A \in \text{lfp}(T_P),$$

where $\text{lfp}(T_P)$ is the least fixed point of $T_P$ and where $\text{comp}(P)$ is Clark’s
completion of program $P$ (cf. [5,11]). This asymmetry is unpleasant since fixed
points of $T_P$ are the Herbrand models of $\text{comp}(P)$. (For a discussion of the


* We refer the reader to [11] or [2] for definitions and discussion pertaining to clauses and logic
 programs that are not otherwise presented in this paper.

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upward and downward transfinite iterations of an operator on a complete lattice, e.g. $T_p$, see \cite{11}.)

It is easy to give examples of programs $P$ for which $T_p \downarrow \omega \neq \text{gfp}(T_p)$:

$$q(a) \leftarrow p(X),$$

$$p(s(X)) \leftarrow p(X).$$

Here

$$T_p \downarrow \omega = \{ q(a) \} \neq \emptyset = T_p \downarrow (\omega + 1) = \text{gfp}(T_p),$$

while

$$T_p \uparrow \omega = \text{lfp}(T_p) = \emptyset.$$ 

We count two previous approaches in the literature that researchers have taken to remedy some of the problems associated with this asymmetry. The first of these is due to Jaffar and Stuckey \cite{9}. They call a definite clause program $P$ with the property that $T_p \downarrow \omega = \text{gfp}(T_p)$ a canonical program and show that every definite clause program $P$ has, in a suitable sense, a conservative extension to a canonical program. Via such extensions the class of canonical programs represents the class of all definite clause programs \cite{8}.

A variation on and an extension of the result of Jaffar and Stuckey is due to Wallace \cite{17}. Although perhaps not apparent at first glance, Jaffar and Stuckey's conservative extension construction is an effective semantics preserving transformation that embeds the class of all definite clause programs into the class of canonical ones. Wallace extends the definition of canonical program to apply to general logic programs in effect using an operator $\Phi_p$ previously studied (in slightly different guises) by Fitting \cite{6} and Kunen \cite{10}. In one guise $\Phi_p$ is seen to be analogous to $T_p$, but is defined on partial Herbrand interpretations using the three truth values \{t, f, u\} (u for undefined). Wallace's definition of canonical program coincides with that of Jaffar and Stuckey for definite clause programs, and he gives an effective semantics preserving transformation that embeds the class of general programs into the class of canonical general programs.

Blair \cite{3} showed that the Jaffar–Stuckey construction can be modified to avoid expanding the Herbrand universe of the program to be transformed.

The second approach fundamentally involves changing the Herbrand universe to another algebra upon which to base structures but which satisfies Clark's equality theory. This is the approach in the proof of a result of Kunen (cf. theorem 6.4, in \cite{10}). Given a general program $P$, Kunen constructs a limit $N$ of a sequence of ultrapowers of the partial Herbrand structures $\Phi_p \uparrow n$, $(n \in \mathbb{N})$. Each of the ultrapowers in the sequence has the same domain which is a reduced direct product $\mathcal{D}_L$ of the Herbrand universe $\mathcal{U}_L$ of the language $L$ of $P$. It can be extracted from Kunen's proof that $T_p^{\mathcal{D}_L} \downarrow \omega = \text{gfp}(T_p^{\mathcal{D}_L})$, i.e., that $P$ is canonical over the algebra $\mathcal{D}_L$. $\mathcal{D}_L$ is determined by $L$, independently of $P$. 