Residual stresses have a considerable effect on the viability of machine components.

These stresses are often measured by cutting rods or rings of rectangular or other cross section, from which layers are removed in sequence by electrolytic etching [1, 2]. However, it is frequently necessary to use planar curved specimens rather than straight rods for this purpose, e.g., specimens cut from parts of turbine or compressor disks, turbine blade roots or edges, etc.

We consider here residual-stress determination for the surface layer of a rod of any cross section. The problem is simplified by assuming that the cross section has a plane of symmetry and that the layers are perpendicular to this. Consider a rod clamped at the end in an x, y coordinate system (the clamp lies at the origin), with the rod described by y(x) or x(y) as in Fig. 1. The ratio of the height h of the section to the minimum radius of curvature R of the rod axis is small (h - R \approx 0.2), i.e., the rod is of low curvature. The residual stresses in a layer at distance a from the surface are constant along the rod, apart from small areas at the ends.

A layer of material of thickness da at a distance a from the surface is removed over a length AB (Fig. 1); removal of da is equivalent to applying a bending moment dM. The rod therefore bends, and the change in the deflection of the free end along the x or y axis is defined by

\[ df_x(a) = \frac{dM}{EI_x(a)} \int_a^b [Y - y(x, a)] \, dx = dM \frac{df_x}{EI_x(a)} K_x(a), \quad (1) \]

or

\[ df_y(a) = -\frac{dM}{EI_y(a)} \int_a^b [X - x(y, a)] \, dy = -\frac{dM}{EI_y(a)} K_y(a), \quad (2) \]

Here \( EI_x(a) \) is the rigidity of the rod after removing layer a and X and Y are the coordinates of the end for the free end:

\[ K_x(a) = Yl_{AB}(a) - S_{xAB}(a); \quad (3) \]

\[ K_y(a) = Xl_{AB}(a) - S_{yAB}(a), \quad (4) \]

where \( l_{AB}(a) \) is the length of part AB of the longitudinal axis after removing layer a and \( S_{xAB}(a) \) and \( S_{yAB}(a) \) are the static moments of parts AB with respect to the x and y axes after removing layer a.

Equations (1) and (2) allow us to use the deflections to determine the residual stresses in etching such a rod; the working formulas are derived here by analogy with the derivation of the formulas for a rectangular rod of arbitrary cross section [1]. The final result is

\[ \sigma_x(a) - \sigma_y(a) = \frac{E}{K_x(a)} \left\{ \frac{l_x(a)}{(h - a - e(a)) b(a)} \right\} \times \]

\[ \times \frac{df_x}{da}(a) - \int_a^b \left[ h - a - e(\xi) + \frac{l_x(\xi)}{F(\xi)(h - d(\xi))} \right] \frac{df_x}{d\xi}(\xi) \, d\xi, \quad (5) \]

Fig. 1. Deformation of a rod on removing layers.

or

\[ \sigma^* (a) - \sigma_a (a) = -\frac{E}{K_y(a)} \left\{ \frac{I_x (a)}{(h - a - \varepsilon (a)) b (a)} \times \frac{df_y}{da} (a) - \int_0^a \left[ h - a - \varepsilon (\xi) + \frac{I_x (\xi)}{F (\xi) (h - \xi - \varepsilon (\xi))} \right] \frac{df_y}{d\xi} (\xi) d\xi \right\}. \]  

where \( \sigma^* (a) \) represents the residual stresses existing in layer \( a \) after removing all the previous layers, \( \sigma_d (a) \) represents the additional stresses in layer \( a \) due to removal of the previous layers, \( F(a) \) and \( F(\xi) \) are the areas of the rod after removal of layers \( a \) and \( \xi \), respectively, and \( I_z (a) \) and \( I_z (\xi) \) are the moments of inertia after removal of layers \( a \) and \( \xi \) with respect to the \( z \) and \( z' \) axes, which pass through the corresponding centers of gravity (see Fig. 1 for other symbols).

In the particular case of the rod of rectangular cross section,

\[ e (a) = \frac{1}{2} (h - a); \quad F (a) = b (h - a); \]

\[ I_x (a) = \frac{b (h - a)^3}{12}; \]

\[ e (\xi) = \frac{1}{2} (h - \xi); \quad F (\xi) = b (h - \xi); \]

\[ I_x (\xi) = \frac{b (h - \xi)^3}{12}. \]

Then

\[ \sigma^* (a) - \sigma_a (a) = \frac{E}{6K_y (a)} \left\{ \frac{1}{2} (h - a)^2 \frac{df_y}{da} (a) + 4 (h - a) f_x (a) + 2 \int \frac{df_y}{d\xi} (\xi) d\xi \right\}. \]  

or

\[ \sigma^* (a) - \sigma_a (a) = -\frac{E}{6K_y (a)} \left\{ \frac{1}{2} (h - a)^2 \frac{df_y}{da} (a) + 4 (h - a) f_y (a) + 2 \int \frac{df_y}{d\xi} (\xi) d\xi \right\}. \]  

A point here is that \( \sigma_d (a) \) in (7) and (8) (the second and third terms) differs somewhat from the true value [1]; this difference is estimated in principle by bringing \( K(a) \) within the integral in (5) and (6). However, in practice, the changes in \( y(x, a) \) and \( x(y, a) \), and consequently in \( K_x (a) \) and \( K_y (a) \), on removing the layers are minor, and usually they can be neglected.

Often, the residual stresses in the surface are appreciably affected by the specimen cutting, particularly if the cross section is of small height \( h < 1.5-2 \text{ mm} \); the residual stresses due to removal of the specimen are thereby substantially increased. Further, the layer removed on the cutting side in electroerosion cutting is \( 0.1-0.2 \text{ mm} \), and this becomes comparable with the height of the specimen, which also increases the residual stresses produced by the cutting itself.

In general, cutting a rod specimen from a component is equivalent to applying a normal force \( N_C \) and a bending moment \( M_C \) (Fig. 2).

Here, the additional stresses due to the cutting \( \sigma_C (a) \) may be determined by calculating the situation for a rod of small curvature deflected in bending in conjunction with tension or compression; the bending may be characterized in terms of the deflection \( f_x \) or \( f_y \) after cutting, while the tension (compression) may be characterized in terms of the relative deflection \( \varepsilon_C \) of the axis. Then