Conical casings are one of the basic elements in rotors for gas turbines, rotors for axial compressors, wheels for centrifugal superchargers, etc. However, engineering methods for designing turbine discs of nonsymmetrical profile are very laborious and complex. Therefore at the present time movement and stresses in such elements are normally calculated with the use of computers.

The use of methods, algorithms, and programs developed earlier for the solution of two dimensional axially symmetrical problems of elastic and plastic deformation of seamless forged rotors [1, 2] permit calculation on the "Ural-2" computer of the stress field and movement for a number of conical discs with regard to the stress concentrations.

The present method assumes that the disc consists of a series of ring shaped elements with a thickness \( z \) and a width \( \Delta r \). The boundaries of the ring are located on the radii \( r_n \). In deriving the equilibrium equations in the final differences it is assumed that at the point under consideration the derivatives from movements along the radius are equal to:

\[
\begin{align*}
\dfrac{d^2U_n}{dr^2} &= \dfrac{U_{n+1} - 2U_n + U_{n-1}}{\Delta r_{n-1,n} + \Delta r_{n,n+1}}; \\
\dfrac{dU_n}{dr} &= \left(\dfrac{U_{n+1} - U_n}{\Delta r_{n,n+1}} - \dfrac{U_n - U_{n-1}}{\Delta r_{n-1,n}}\right) \dfrac{2}{\Delta r_{n-1,n} + \Delta r_{n,n+1}}.
\end{align*}
\] (1)

After substituting Eq. (1) in the corresponding differential equation for equilibrium, for any point \( n \) we obtain a condition of equilibrium for this point in final form:

\[
a_{n-1,n}U_{n-1} - \left(a_{n-1,n} + S_n + a_{n,n+1}\right)U_n + a_{n,n+1}U_{n+1} = R_n.
\] (2)

If equations similar to Eq. (2) are set up for all points on the boundaries of the ring elements, we obtain a system of linear algebraic equations expressing the relationship of the radial movements \( U_n \) to the loads \( R_n \):

\[
\begin{align*}
- (S_1 + a_{1,2})U_1 + a_{1,2}U_2 &= R_1; \\
(1,2) & \\
a_{1,2}U_1 - (S_2 + a_{2,3})U_2 + a_{2,3}U_3 &= R_2; \\
a_{2,3}U_2 - (S_3 + a_{3,4})U_3 + a_{3,4}U_4 &= R_3; \\
& \vdots \\
a_{n-1,n}U_{n-1} - (a_{n-1,n} + S_n)U_n &= R_n,
\end{align*}
\] (3)

where

\[
\begin{align*}
a_{n-1,n} &= \left(\dfrac{\psi\Delta r}{2r_n}\right)_{n-1,n} + \left(\psi\Delta r\right)_{n,n+1}, \\
S_n &= \left(\dfrac{\psi\Delta r}{2r_n}\right)_{n,n+1} - \left(\psi\right)_{n,n+1}.
\end{align*}
\]

* For simplicity these will be called conical discs.
Fig. 2. Stress distribution on the inside surface of conical discs with hubs (a) and without hubs (b) (assumed speed $n = 10,000 \text{ rpm}$).

is the rigidity of the ring elements:

$$\psi = \frac{E}{1 - \mu^2}; \quad \psi = \frac{\mu E}{1 - \mu^2}.$$  

Solution of the system of Eqs. (3) permits us to determine the distribution of movements along the radius of the disc $U_1, U_2, \ldots, U_n$ under the action of the loads $R_1, R_2, \ldots, R_n$. Then the stress in the center portion may be calculated by the formulas

$$\sigma_{n,n+1} = \left(\frac{\psi}{2r}\right)_{n,n+1} (U_{n+1} - U_n) + \left(\frac{\psi}{2r}\right)_{n,n+1} (U_{n+1} + U_n) - (\beta)_{n,n+1};$$

$$\sigma_{n,n+1} = \left(\frac{\psi}{2r}\right)_{n,n+1} (U_{n+1} + U_n) + \left(\frac{\psi}{2r}\right)_{n,n+1} (U_{n+1} - U_n) - (\beta)_{n,n+1}. \quad (4)$$

Here $\beta = \alpha E / (1 - \mu)$ is the temperature coefficient of hydrostatic pressure.

The system of linear algebraic Eqs. (3) is solved by the Gauss method. The method for solving the elastoplastic problem consists of repeated solutions of the elastic problem with variations of the parameters $E$ and $\mu$, depending upon the degree of deformation of the material.

A study was made of several variations in the design of conical discs, differing in the angle of the generatrix of the blade and the presence or absence of a hub (Table 1). The first and second groups of calculations (problems 1-8) were done for discs with hubs and with blade generatrix angles of 0, 5, 10, and 20°. The speeds used in the calculations for these problems are shown in Table 1. In the third and fourth groups of calculations (problems 9-17) discs without hubs were studied. Here the blade generatrix angles were 0, 5, 10, 20, and 30°.

The discs had the following dimensions: outside diameter 245 mm; inside diameter of discs with hubs 30 mm; outside diameter of the hub 60 mm; width of the hub 65 mm; radius of the joint between the blade and the hub 10 mm; inside diameter of discs without hubs 70 mm; thickness of the blade 20 mm.

All of the calculations were made for discs made of steel with a tensile strength of 50 kg/mm² and an elongation of 5%. In the calculations for flat discs one quarter of the cross section was considered since the stress field of such a disc is symmetrical relative to two mutually perpendicular axes $r$ and $z$. Calculations for discs with an inclined blade were made for a symmetrical (relative to the radius) half of the disc.

For the first two groups of calculations of nonuniform division of the discs into elementary blocks $\Delta r$ (along the radius) and $\Delta z$ (along the axis $z$) was used, making it possible to consider the concentration of stresses dependent on the actual geometry of the junction of the disc blade with the hub. In the problems of the later groups the division along the radius was uniform but along the axis $z$ nonuniform. A comparison of the division used into blocks with the actual profile of the discs is shown to be a good approximation of the boundaries of the area being studied.

As a result of the calculations two dimensional fields were obtained of the radial ($\sigma_r$), tangential ($\sigma_t$), axial ($\sigma_z$), and ($r_{rz}$) stresses, and also the field of movements in elastic and elastoplastic deformation. Figure 1 shows the distribution of the fields of elastoplastic stresses for discs with the angle $\beta = 10^\circ$.