CERTAIN QUESTIONS OF THE DYNAMICS OF FRACTURE

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It is known that in the stages of design, production, and service of a structure it is necessary to take into consideration not only the possibility of occurrence of defects and cracks but also the features of their development. To prevent failure of structures, i.e., to retard and arrest crack development, it is necessary to know the rules of this process.

Let us consider unstable crack propagation under conditions of the absence of plastic microdeformations and in the occurrence of local flow close to a crack tip. Such a process of crack development does not require additional external energy, and the rate of failure is a value less than the speed of sound in the given material.

By mathematical analysis it has been established that, in contrast to the problem of a stationary crack in a stressed continuous medium, in this case at the crack tip there occurs a field of stresses with the same form of singularity \( r^{-1/2} \) as in the case of a stationary crack but significantly dependent upon the rate of crack propagation. In fracture mechanics this field and its rate relationships are determined by the dynamic stress intensity factor.

In advance of a crack in an infinite medium the dynamic stress intensity factor is described by the equation

\[
K_I(a, \dot{a}) = k [a(t)] K_s[a(t)]
\]

where \( K_s(a) \) is the stress intensity factor for a suddenly arrested crack with a length of \( a \), which in value is close to the static factor for the given crack length and boundary conditions, and \( k \) is a function dependent upon the rate of crack development, Poisson's ratio \( \nu \), and, in general, upon the character of the boundary conditions. Considering this function as the ratio of the dynamic and static stress intensity factors it may be stated that it decreases with an increase in crack propagation rate and reaches zero with \( a = c_R \) [1, 2], where \( c_R \) is the speed of Rayleigh waves, which in practice is not the limiting theoretical value realized. However, if the dynamics of a crack in a body of finite dimensions is considered, then it must be kept in mind that the relaxation of the field of stresses around a propagating crack is determined by the elastic waves of stresses, i.e., by the longitudinal, transverse, main, and Rayleigh, which propagate with corresponding rates and in a corresponding manner (Fig. 1). The reflections and interactions of these waves with the boundaries of the body may change the stressed state with time both as the result of temporary changes in the energy constituent, which is the work of the boundary forces \( E_B \), and as the result of the possible influence of the reflected waves on the stress at the crack tip (Fig. 2).

In essence, here we encounter the subsequent dynamicness of the problem. The dynamic state of the stresses at the crack tip must be taken into consideration, first, when a force impulse or an impulse of stresses of certain parameters is applied to a stationary crack (Fig. 3) and, second, when in an infinite medium the crack propagates at a certain rate \( \dot{a} \) and the field of stresses occurring is a function of the rate \( \dot{a} \). In a body of finite dimensions with a developing crack there occurs a combination of both cases and for the dynamic stressed condition at a crack tip in the majority of cases it is necessary to consider the stress intensity factor \( K_I(a, t) \) and its critical value \( K_{ID} \) (Fig. 2). It should be noted that taking into consideration the dynamicness in relation to the boundary conditions and the conditions of
Fig. 1. The emission of elastic stress waves by the tip of a propagating crack: 1) transverse wave with a speed of \(c_2\); 2) longitudinal wave with a speed of \(c_1\) or \(c_3\); 3) main wave with a speed of \(c_2\); 4) Rayleigh wave with a speed of \(c_R\).

Fig. 2. Schematic representation of the influence of elastic waves on the stress at the crack tip in a body of finite dimensions.

loading in all of these cases may significantly change the stress intensity factor in comparison with that in the static state.

We present the mathematical description of a system with a developing crack on the basis of the equations of movement for an elastic isotropic medium:

\[
(\lambda + \mu) \frac{\partial e}{\partial t} + \mu \nabla^2 u + F_i = \rho \frac{\partial u_i}{\partial t} \quad (i = 1, 2, 3)
\]

(2)

and the equations of the balance of the rates of change in the constituent energies participating in the failure process:

\[
\frac{dE_A}{dt} = \frac{dE_{el}}{dt} + \frac{dE_{kin}}{dt} + \frac{dE_{dis}}{dt}.
\]

(3)

where \(e\) is the volumetric deformation, \(u_i\) is the constituents of the vector of displacements, \(F_i\) is the constituents of the volumetric forces, \(\rho\) is the density of the material, \(t\) is time, \(\lambda\) and \(\mu\) are the Lame constants, \(E_A\) is the work of the external forces, \(E_{el}\) is the elastic constituent of the stored energy, \(E_{kin}\) is the kinematic energy of the system, and \(E_{dis}\) is the sum of all of the irreversible constituents of the energy at failure. The individual components of the energy changes may be found from the following equations if the displacements and their derivatives making up the stresses \(\varepsilon_{ij}\) and the external forces \(T_{ij}\) are known:

\[
\dot{E}_A = \int_V \sum_{i=1}^3 T_i \dot{u_i} dV + \int_S \sum_{i=1}^3 F_i \dot{u_i} dS,
\]

\[
\dot{E}_{el} = \int_V \sum_{i=1}^3 \varepsilon_{ij} \ddot{e}_{ij} dV; \quad E_{kin} = \int_V \rho \sum_{i=1}^3 \dot{u_i} \ddot{u_i} dV,
\]

(4)

where \(\varepsilon_{ij}\) are components of deformation, \(V\) is volume, and \(S\) is the surface of the body.

From the equations presented it may be seen that obtaining an analytical solution of the problem for a body of finite dimensions is very difficult, and therefore it is necessary to use numerical methods. At the same time, the problems occurring with the use of these methods in dynamics, particularly the introduction of artificial anisotropy and wave filters, the simulation of movement of the end of the crack, etc., must be taken into consideration.

To obtain reliable results [3, 4] in each individual case it is necessary to know the properties and capabilities of the numerical model. The difficulties occurring with the use of such an approach served as the reason for the search and other means of solution of the