The reinforcing fibers employed for strengthening composition materials vary substantially in strength [1]. The passage of a group or bundle of parallel fibers into the limiting state will therefore be a gradual process, passing through a succession of quasiequilibrium states corresponding to a specific value of the load. In order to analyze this peculiar behavior of composition materials [2], involving the rupture of the weakest reinforcing elements, we must consider the rupture of a bundle of parallel fibers of different tensile strengths.

Quasiequilibrium States of Bundles of Fibers. We shall consider that the fibers are statistically uniform, having equal diameters and no rheonomic properties. We shall neglect effects associated with the twisting (torsion) of the fibers and interfiber friction. We consider the bundle of fibers as a system discrete in the statistical sense [7, 8].

Allowing for the foregoing considerations, the load acting on a fiber when the bundle undergoes a strain $\varepsilon$ may be expressed in the form

$$s = s(s, a_1, \ldots, a_n),$$

where $a_1, \ldots, a_n$ are parameters respectively defining the strain diagram specific to each fiber.

Rupture occurs at a load of

$$s_f = s(\varepsilon_f, a_1, \ldots, a_n).$$

For a bundle comprising a large number of fibers $N$ of length $l$, the probability that the breaking strain $\varepsilon_f$ and the parameters $a_1, \ldots, a_n$ for a particular fiber will occur within specified close limits may be written as

$$\varphi_f(\varepsilon_f, a_1, \ldots, a_n) \, d\varepsilon_f \, da_1 \ldots da_n,$$

and the total stress on the bundle $Q$ when the deformation $\varepsilon$ is given by

$$Q(\varepsilon) = N \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} s(s, a_1, \ldots, a_n) \, d\varepsilon_f \, da_1 \ldots da_n.$$  

The rupture of the bundle occurs at a strain $\varepsilon_{cr}$, which may be found by maximizing the extremum of Eq. (4)

$$\frac{dQ(\varepsilon)}{d\varepsilon} \bigg|_{\varepsilon=\varepsilon_{cr}} = 0.$$  

The maximum (breaking or rupturing) stress (load) is then

$$Q_{\text{max}} = Q_f = Q(\varepsilon_{cr}).$$

For mineral fibers, possessing only a slight nonlinearity of the strain diagrams, the dispersions of the parameters $a_1, \ldots, a_n$ are small compared with the dispersion of $\varepsilon_f$, i.e., we may consider that the strain diagram is defined by two parameters, $\varepsilon_f$ and $a$. Then Eq. (3) will take the form

$$\varphi_f(\varepsilon_f, a) \, d\varepsilon_f \, da,$$  

and the probability density for the quantity \( \varepsilon \) will be

\[
 f(\varepsilon) = \int_0^\infty \varphi_\varepsilon(\varepsilon, a) \, da,
\]

and the load on the fiber \( s(\varepsilon, a) \).

For a load \( Q \) some of the fibers rupture and the load is taken up by the remaining \( r \) fibers. For a large \( N \) the probability of the survival (nonrupture) of \( r \) fibers will be \( r/N \), while the number of unbroken fibers will be

\[
 r = N \int_0^\infty f(\varepsilon) \, d\varepsilon = \int_0^\infty \varphi_\varepsilon(\varepsilon, a) \, da.
\]

The stress on the bundle \( Q(\varepsilon) \) is written in the form

\[
 Q(\varepsilon) = N \int_0^\infty \varphi_\varepsilon(\varepsilon, a) \, da.
\]

Equation (5) takes the form

\[
 \frac{d}{d\varepsilon} \int_0^\infty s(\varepsilon, a) \varphi_\varepsilon(\varepsilon, a) \, da \bigg|_{\varepsilon = 0} = 0.
\]

For elastic fibers with similar curves \( s(\varepsilon) \) we may take \( s(\varepsilon, a) = a(\varepsilon) \), and then the relation (8) will transform in the following manner:

\[
 Q(\varepsilon) = N a(\varepsilon) \int_0^\infty \varphi_\varepsilon(\varepsilon, a) \, da.
\]

If \( \varepsilon_f \) and \( a \) are independent, then \( \varphi_{\varepsilon_f}(\varepsilon_f, a) = \varphi(\varepsilon_f) \varphi(a) \), and, taking \( a(\varepsilon_f) = a \), we have

\[
 Q(\varepsilon) = na(\varepsilon_f) \int_0^\infty \varphi_\varepsilon(\varepsilon_f) \, da.
\]

Condition (5) is the written in the form

\[
 \frac{d}{d\varepsilon} \left( \int_0^\infty \varphi_\varepsilon(\varepsilon_f) \, da \bigg|_{\varepsilon = 0} \right) = 0.
\]

For elastic fibers we may convert to the probability density of the breaking load. In this case relation (8) transforms:

\[
 Q(\varepsilon) = N s(\varepsilon) \int_0^\infty g_\varepsilon(s) \, ds.
\]

On applying condition (5) to the stresses, the load at the instant of reaching the limiting state may be determined from

\[
 \frac{d}{ds} \left( \int_0^\infty g_\varepsilon(s) \, ds \bigg|_{s = s_{cr}} \right) = 0.
\]

The condition for the quasiequilibrium state of the bundle of fibers has the form

\[
 Q(\varepsilon) < Q_{\varepsilon_f} = \left[ \int_0^\infty g_\varepsilon(s) \, ds \bigg|_{s = s_{cr}} \right]_{\max} = Q_{s = s_{cr}}.
\]

A scheme illustrating the possible existence of quasiequilibrium states and the progressive rupture of the fibers is presented in Fig. 1.

If we denote \( a(\omega) = \alpha(1/\omega) \) as the number of breaking fibers (the "damage" suffered by the bundle), then the total load, the load on the fiber, and the damage will be related by

\[
 Q = N s \int_0^\infty g_\varepsilon(s) \, ds = N \frac{1 - \alpha(\omega)}{\omega},
\]