THE STATE OF STRESS OF AN ELASTICALLY DEFORMED HELICAL HEAT EXchanger WITH A RIGID CORE AND FLAT TUBULAR GRIDS

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A helical heat exchanger with a rigid core (a design which was suggested in [1, 2]) has less metal for the same equivalent heat-exchange surface than shell-and-tube heat exchangers of other types. This is particularly true of large heat exchangers used for gas separation in the heavy chemical industry.

The helical heat exchanger (Fig. 1) consists of an outer cylindrical casing with a lid and base, two tubular grids, and a core on which the heat-exchange tubes are wound. To increase the supporting capacity of the flat tubular grids, the core is welded to them — i.e., it plays the role of an anchor. The supporting capacity of the heat-exchange tubes can be neglected.

Plastic deformation is impermissible in making a heat exchanger from steel that becomes brittle at low temperatures, and the design must be based on detailed consideration of the stresses arising at the joints between components. We therefore examined the state of axial stress and deformation in a statically indeterminate system of intersecting elastic thin plates and cylindrical shells in a helical heat exchanger. The system was stressed by internal pressure and by temperature gradients in the components.

Figure 2 shows how the five main parts of the exchanger are connected: the body, the intermediate jacket cylinder, the central cylinder, and the tube grid, which consists of two parts (annular plate and disk) differing in cylindrical rigidity, because the annular plate is weakened by the holes under the tubes.

Allowance for Static Indeterminacy. This arises from indeterminacy in the distribution of the axial forces between the body and the central cylinder, set up by the pressure in the tube grids. A third constant of integration therefore appears in the expression for the axial force in a long cylindrical jacket cylinder [3]:

\[ N_1 = \frac{pR}{2} + \frac{B_2}{R} \]  

(1)

This gives expressions for the axial displacement

\[ u = \frac{L}{2E_s} \left[ (1 - 2\nu) \frac{pR}{2} + \frac{B_2}{R} \right] \]  

(2)

and for the radial displacement

\[ w = (B_1 \sin \alpha + B_2 \cos \alpha) \sigma - B_2 \alpha + (2 - \nu) \frac{\rho R}{2E_s} - \frac{vB_2}{E_s} \]  

(3)

The indeterminacy does not affect the analytic expressions for the angles of rotation \( \theta \), bending moments (axial \( M_1 \) and circumferential \( M_2 \), and the circumferential forces \( N_2 \) for cylindrical jackets.

Effect of Temperature Differences. In the case of a thin-walled cylindrical shell (with allowance for thermal insulation) the temperature differences along the walls can be neglected. The additional displacements \( u_t \) in the axial and \( w_t \) in the radial directions are

\[ u_t = \frac{\gamma L}{2}; \quad w_t = \gamma R, \]  

(4)

where \( \gamma \) is the linear expansion coefficient and \( t \) is the temperature.
The tubular grid bounds heat-exchanging media with different temperatures; therefore we must allow for the temperature difference across its thickness. For thin plates we can approximately use a linear depthwise temperature distribution:

\[ t(x) = kx + t_{av}. \]

where

\[ k = -\frac{t_1 - t_4}{3} ; \hspace{1cm} t_{av} = \frac{t_1 + t_4}{2}. \]

The temperature drop (5) over the tubular grid affects only the expressions for the radial bending moments \( M_r \) and the peripheral bending moments \( M_\theta \); it does not affect the expressions for the deflections \( w \), the angles of rotation \( \theta \), and the transverse force \( Q \) (cf. formula (12)).

**Initial Equations.** Let us write down expressions for the unknowns in terms of the internal pressure, taking account of the static indeterminacy and the influence of temperature.

For the casing

\[ w = (B_1 \sin \alpha e + B_2 \cos \alpha e) e^{-\alpha \xi} + (2 - \nu) \frac{pR_s}{2Es} - \frac{\nu B_3}{Es} + \gamma t_R; \]

\[ u = \frac{L}{2Es} \left[ (1 - 2\nu) \frac{pR_s}{2} + \frac{B_3}{R} \right] + \gamma L; \]

\[ \theta = \frac{\xi}{R} \{(B_1 - B_2) \cos \alpha e - (B_1 + B_2) \sin \alpha e\} e^{-\alpha \xi}; \]

\[ M_1 = 2D \left[ \frac{\xi}{R} \right] (B_1 \cos \alpha e - B_2 \sin \alpha e) e^{-\alpha \xi}; \hspace{1cm} M_\theta = \nu M_1; \]

\[ N_1 = \frac{pR_s}{2} + \frac{B_2}{R}; \hspace{1cm} N_\theta = \frac{E_s}{R} (B_1 \sin \alpha e + B_2 \cos \alpha e) e^{-\alpha \xi} + pR, \]

where

\[ \xi = \sqrt[3]{3(1 - \nu)} \sqrt{\frac{R}{s}}; \hspace{1cm} D = \frac{Es}{12(1 - \nu)}; \hspace{1cm} \alpha = \frac{\xi}{R} \]

(\( \xi \) is the axial coordinate for a cylindrical shell).

For the core

\[ w_\varepsilon = (K_1 \sin \alpha e + K_3 \cos \alpha e) e^{-\alpha \xi} - (2 - \nu) \frac{pR_s}{2Es} - \frac{\nu K_3}{Es} \gamma \xi R_\varepsilon; \]

\[ u_\varepsilon = \frac{L}{2Es} \left[ (1 - 2\nu) \frac{pR_s}{2} + \frac{K_3}{R_\varepsilon} \right] + \gamma L; \]

\[ \theta_\varepsilon = \frac{\xi}{R_\varepsilon} \{(K_1 - K_3) \cos \alpha e - (K_1 + K_3) \sin \alpha e\} e^{-\alpha \xi}; \]