INVESTIGATION OF THE STRESS STATE OF
THICK MULTIPLY CONNECTED PLATES BY THE
FINITE ELEMENT METHOD

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In [1] an expression is obtained for the variation of the energy functional for a nonorthogonal curvi-
linear basis coordinate system and an arbitrary local coordinate system. The equations of the finite ele-
ment method (FEM) set up on the basis of this expression realize curved isoparametric three-dimensional
finite elements.

In order to investigate thick rectangular plates and massive bodies referred to a Cartesian basis co-
ordinate system, we must simplify the expression of the variation of an elastic potential (6) presented in
the report [1]. As a result of this, certain tensor quantities are altered:

\[ \begin{align*}
\bar{e}_{mn} &= e_{mn}; \\
\bar{g}_{mn} &= g_{mn}; \\
\bar{b}_{m} &= b_{m}; \\
\bar{a}_{k} &= a_{k}; \\
\bar{r}_{k} &= r_{k}; \\
\bar{\Gamma}_{k} &= \Gamma_{k} = 0.
\end{align*} \]

Here and below all symbols correspond to those used in [1, 2].

After tensor transformations of expression (6) the equation of variation of the elastic potential for
the basis and the local Cartesian coordinate systems is represenfed in the form

\[ \delta W = \frac{1}{288} \left( \sum_{l=1}^{3} \sum_{m=1}^{3} \sum_{n=1}^{3} u_{l(m)'} \sum_{s=1}^{3} \sum_{r=1}^{3} u_{r(n)'} \sum_{l=1}^{3} (s_{l(m)} r_{l(n)} + 3) \times \left( \delta_{l} \left[ s_{l(m)} r_{l(n)} + \frac{\lambda}{\mu} \delta_{l(m)} r_{l(n)} \right] + \frac{3}{3} \right) \right), \]

where \( \delta W \) is the variation of the energy functional of the element;

\[ d_{i} = \begin{cases} 1 & \text{for } i \neq j; \\ 0 & \text{for } i = j. \end{cases} \]

The FEM set up on the basis of expression (2) was verified by solving a test problem — the flexure
of a thick rectangular plate subjected to a uniformly distributed load (Fig. 1a). The data for the calcula-
tion is as follows: \( a = 100 \text{ cm}, b = 100 \text{ cm}, h = 20 \text{ cm}, \nu = 0.25, E/q = 50.0 \). The grid for dividing the plate
into finite elements is \( 5 \times 7 \times 7 \) (the first figure indi-
cates the number of divisions along the coordinate line \( \alpha_{1} \), the second along \( \alpha_{2} \), and the third along \( \alpha_{3} \)).

The results of the solution were compared with the results of the solution by the defining states method
(DSM) [3] (Fig. 1b). In Table 1 we have shown the com-
parison of the displacements of the central point. From
the data of this table we see the agreement of the results

<table>
<thead>
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<th>( \frac{a}{h} )</th>
<th>DSM</th>
<th>FEM</th>
<th>Error, %</th>
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<td>5.459</td>
<td>5.298</td>
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<td>-1</td>
<td>5.414</td>
<td>5.279</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Fig. 1. Solving the problem of flexure of a thick rectangular plate under the action of a distributed load.

Fig. 2. Calculation schemes of thick rectangular plates: □) \( \sigma^{i2} \); ■) \( \sigma^{i3} \); △) \( \sigma^{23} \); ▲) \( \sigma^{22} \); +) \( \sigma^{33} \).

The results of the calculation are presented in the form of curves of the components of the stress tensor across the section made along the coordinate line \( \alpha^3 \) (Fig. 3), and constant deflection lines (Fig. 4) for the problem with two cutouts. In Fig. 2 we have marked the maximum values of the components of the stress tensor by dots.

An analysis carried out allows us to draw the conclusion that the addition of two cutouts with clamped points to the plate with two cutouts with partially clamped edges does not influence the distribution of the maximum values of the components \( \sigma^{22} \), \( \sigma^{33} \) and the shear stresses \( \sigma^{13} \); however, the maximum values of the shear stresses \( \sigma^{12} \) are displaced closer to the clamped points, while the shear stresses \( \sigma^{23} \) are displaced closer to the clamped outer planes.