**Dispersion in non-Newtonian fluids: effects of chemical reaction**

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With 2 figures and 6 tables

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**Notation**

- $c$: concentration of the solute in the fluid
- $\frac{dP}{dz}$: pressure gradient
- $D$: molecular diffusion coefficient
- $D^*$: equivalent dispersion coefficient
- $J^*$: flux used in Fick's law of diffusion
- $K$: homogeneous chemical reaction rate constant
- $L$: typical length of the system
- $m$: consistency of the power law fluid
- $n$: flow behaviour index (power law index)
- $p$: index for systems
- $\bar{Q}$: average flux of the solute across a section of the duct
- $R$: radius of the tube and half thickness of the channel
- $(r,z)$: coordinate system
- $t$: time
- $v$: velocity component of the fluid along z-direction
- $\bar{v}$: mean velocity of the fluid along z-direction
- $\tau$: shear stress
- $\tau_o$: yield stress
- $\mu$: consistency of the fluid (in Bingham and Casson models)

**1. Introduction**

The dispersion of a solute matter in fluid flowing through channels and pipes has important applications in chemical as well as biological systems. One of the simplest way to study the dispersion through a fluid has been suggested by Sir Geoffrey Taylor (1). By considering the dispersion of a solute matter in a solvent flowing through a tube, he has shown that, relative to a plane moving with the average speed of the flow, the solute diffuses with an effective (equivalent) dispersion coefficient which depends upon the radius of the tube, the mean velocity and the molecular diffusion coefficient. He has also found the condition under which dispersion of a solute in a solvent can be used to measure the molecular diffusion coefficient (1, 2).

Later, Aris (3) studying the same problem removed the condition imposed by Taylor in his analysis. Since then several investigators have studied the dispersion through Newtonian fluids by considering different situations such as homogeneous and heterogeneous chemical reaction under steady and unsteady conditions, Katz (4), Walker (5), Solomon et al. (6), Gupta et al. (7), Gill et al. (8, 9), Scherer et al. (10).

The first attempt to study dispersion through a non-Newtonian fluid was made by Fan and Hwang (11) in the case of a power law fluid using Taylor's approach (1). Later, Fan and Wang (12) also studied the dispersion through Bingham plastic and Ellis model fluids (13). The dispersion through Reiner-Philippoff model and Eyring model fluids have been studied by Ghoshal (14) and Shah et al. (15) respectively. Prenosil et al. (16) has investigated the dispersion through a power law fluid by considering Aris' approach (3). Nigam and Vasudeva (17) have studied the diffusion and reaction in tubular reactor with non-Newtonian laminar flow.

In the above mentioned investigations the effects of chemical reaction in non-Newtonian fluids have not been taken into consideration. However such effects have been studied in the case of dispersion through Newtonian fluids, among others, by Gupta et al. (7). Keeping these in view, in this paper, we study the effects of homogeneous first-order chemical reaction of a solute in non-Newtonian fluids flowing through parallel plates and circular tubes by considering the following models: (i) Power law model, (ii) Bingham model, (iii) Casson model.

**2. Basic equations**

Consider the symmetrical flow of a non-Newtonian fluid flowing under a constant pressure gradient between two parallel plates separated by a distance $2R$ or through a circular tube of radius $R$, as shown in figure 1.

The constitutive equation for non-Newtonian fluids may be written as (13, 19)
Fig. 1. Dispersion through a fluid in a duct

\[ \left( -\frac{dv}{dr} \right) = f(\tau), \]

where \( \tau \) is the shear stress and \( dv/dr \) is the shear rate and \( f(\tau) \) is a general function prescribed for a given fluid model.

Considering a simple force balance on a cylindrical element of the fluid in the duct we have

\[ \tau = \frac{r}{p+1} \left( -\frac{dP}{dz} \right), \]

and

\[ \tau_R = \frac{R}{p+1} \left( -\frac{dP}{dz} \right), \]

which gives

\[ \frac{\tau}{\tau_R} = \frac{r}{R}, \]

where \( \tau_R \) is the shear stress at the wall \( r = R \), \( p = 0 \) for flow between parallel plates and \( p = 1 \) for flow through a circular tube.

Integrating eq. [1] and using \( v = 0 \) at \( r = R \), we get

\[ v = \int \frac{1}{r} f(\tau) \, dr = R \int \frac{1}{r} f(\tau) \, dy, \]

where \( y = r/R \) and \( f(\tau) \) is a function of \( y \) (see eq. [2]).

The average velocity \( \bar{v} \) is defined as,

\[ \bar{v} = \frac{1}{(2\pi)^p v dr} \int \frac{2\pi^p v dy dr}{R} = \int \frac{(2\pi)^p v(y) dy}{R}, \]

which on using eq. [5] gives,

\[ \bar{v} = \frac{2^p R}{p+1} \int_0^1 \left( y^{p+1} f(\tau) \right) dy, \]

and

\[ v - \bar{v} = \bar{v} g_p(y), \]

where

\[ g_p(y) = \frac{\int_0^1 f(\tau) \, dy}{2^p R \int_0^1 y^{p+1} f(\tau) \, dy} - 1. \]

Assuming that the solute while diffusing, undergoes an irreversible chemical reaction in the fluid, the equation governing the concentration \( c \) of the solute is given by

\[ \frac{\partial c}{\partial t} + v \frac{\partial c}{\partial z} = D \frac{\partial}{\partial r} \left( r^p \frac{\partial c}{\partial r} \right) - kc, \]

where \( D \) is the constant molecular diffusion coefficient and \( k \) is the first order homogeneous chemical reaction rate constant.

Following Taylor (1) eq. [10], relative to a plane moving with the mean speed of the flow, can be written as \((\bar{z} = z - \bar{v} t)\)

\[ \frac{\partial c}{\partial \bar{t}} + \bar{v} \frac{\partial c}{\partial \bar{y}} = \frac{D}{r^p} \frac{\partial}{\partial r} \left( r^p \frac{\partial c}{\partial r} \right) - kc. \]

Introducing the following non-dimensional variables,

\[ \bar{t} = \frac{t}{\bar{t}}, \quad \bar{y} = \frac{L}{y}, \quad \bar{x} = \frac{x}{\bar{y}}, \quad \bar{y} = \frac{r}{R}, \]

eq. [11] can be transformed, after using eqs. [8] and [12], as

\[ \frac{1}{\bar{y}^p} \frac{\partial}{\partial \bar{y}} \left( \bar{y}^p \frac{\partial c}{\partial \bar{y}} \right) = \alpha^2 c - \frac{D}{R^2 \bar{y}^p} \frac{\partial}{\partial \bar{x}} \left( \bar{y}^p \frac{\partial c}{\partial \bar{x}} \right) - kc. \]

Assuming that the Taylor's limiting condition is valid (1), i.e. the partial equilibrium over any cross section of the duct is established, eq. [13] can be approximated as follows

\[ \frac{1}{\bar{y}^p} \frac{\partial}{\partial \bar{y}} \left( \bar{y}^p \frac{\partial c}{\partial \bar{y}} \right) - \alpha^2 c = \frac{R^2 \bar{v}}{D \bar{L}} \frac{\partial}{\partial \bar{x}} g_p(y), \]

where \( \alpha^2 = (k R^2) / D \) and \( \partial c / \partial x \) is a constant with respect to \( y \).

Since there is no chemical reaction at the wall, the boundary condition for concentration \( c \) can be written as,

\[ \frac{\partial c}{\partial \bar{y}} = 0 \text{ at } \bar{y} = 0, \quad \frac{\partial c}{\partial \bar{y}} = 0 \text{ at } \bar{y} = 1. \]