The problem of elastoplastic deformation of a sphere under uniform pressure has been considered first by Saint-Venant [1]. His solution was based on the theory of plastic flow. The same problem was solved by Il'yushin [2] with the aid of the strain theory of plasticity. With the aid of an arbitrary rule of hardening, he analyzed two limiting cases: a completely elastic and a completely plastic sphere. Il'yushin also proved the theorem of simple loading in the case of isothermic loading. The problem of thermoplasticity for a hollow sphere whose materials possesses elasto-stiff-plastic properties has been solved on the basis of the strain theory of plasticity in [3]. The theorem of simple loading for nonisothermic forces has been proved in [4].

Here we consider a closed, hollow sphere exposed to radiation. It is assumed that its outside surface is heated to a definite temperature which remains constant in the course of deformation, while its inside surface temperature remains unchanged. In such a case, a linear temperature distribution is established in the sphere thickness. The sphere boundaries are free of stresses.

We have investigated temperature fields that allow the appearance of plastic deformation regions. The elastoplastic problem is solved on the basis of the strain theory of plasticity of Il'yushin.

The following assumptions have been made: the change in volume is purely elastic, the loading is simple, no strain anisotropy is present; the hardening of the sphere material assumed to be linear, and the elastic and thermophysical properties are assumed to be temperature independent.

Derivation of Basic Relations. Before solving the elastoplastic problem it is necessary to consider the change in elastic and plastic material properties as a result of exposure to radiation. Many experiments indicate that radiation exposure has little effect on such characteristics as the elasticity modulus and Poisson ratio. The ultimate strength and limit of elasticity depend on the total neutron flux and can increase by a factor of more than two which has a significant effect on the stress–strain state.

In case of spherical bodies, the scattering of an electron flux along a radial coordinate can be written as [5]

\[
N_\rho = N \frac{\rho^2}{\rho^*} e^{-\mu(\rho^{-1})},
\]

(1)

Here N is the radiation intensity at the outside surface of the sphere; \( \rho = r_1 / r_0 \) is the ratio of outside and inside radii; \( \rho = r / r_0 \) is the dimensionless running radius (\( r, r_1, \) and \( r_0 \) are the running, outside, and inside radii, respectively); \( \mu \) is the macroscopic effective cross section; \( \mu = \bar{\sigma} \Lambda_0 \bar{\rho} / A \), where \( \bar{\sigma} \) is the effective cross section as referred to one nucleus; \( \Lambda_0 \) is the Avogadro number; \( \bar{\rho} \) is the material density; and A is the atomic weight.

Assuming the hypothesis that the properties at depth \( \rho \) are the same as the properties of the material exposed to radiation of intensity \( N_\rho \), the change in the limit of elasticity with sphere thickness can be analytically written as

\[
\sigma_y = \sigma_y \left[ N \frac{\rho^2}{\rho^*} e^{-\mu(\rho^{-1})} \right],
\]

(2)

where \( \sigma_y \) is the dimensionless yield limit.

In solving the elastoplastic problem for a sphere exposed to radiation it is thus possible to apply the well-known strain theory of plasticity of Il’yushin. In particular, we have to determine the loads at which plastic deformations start to appear, since because of the variation of the elastic limit with thickness we in fact have to consider elastoplastic deformation of a nonhomogeneous body.

The problem is solved in stresses. The well-known treatment of Il’yushin [2] is here extended to the case of temperature effects.

In case of polar symmetry we have one equilibrium equation:

\[ \frac{d\sigma_p}{dp} + \frac{2}{p} (\sigma_p - \sigma_\varphi) = 0. \]  
(3)

Here \( \sigma_p = 2\sigma_p^* / \alpha T_C E(1 - 2\nu) \) are dimensionless radial and tangential stresses, where \( \sigma_p^* \) and \( \sigma_\varphi^* \) is the radial and tangential stress, respectively; \( \alpha \) is the coefficient of linear thermal expansion; \( T_C \) is the outside surface temperature; and \( E \) and \( \nu \) is the elasticity modulus and Poisson ratio, respectively, in the first linear region of the stress-strain diagram.

In general, the uniform pressure on the outside and inside sphere surfaces can be considered given:

\[ \sigma_p = -q; \sigma_\varphi = -q_\varphi \text{ for } \rho = 1; \rho = \rho. \]  
(4)

The solution of the heat equation has the form

\[ \theta(p) = \frac{\rho - 1}{\rho - 1} \frac{p}{p} \times z, \]  
(5)

where \( \theta = T / T_C \) is a dimensionless temperature and \( z = T_C / 200^\circ \).

For a linearly hardening material, the stress-strength relations can be written as

\[ \sigma_p = \frac{2(1-\omega)}{(1-2\nu)(1-\nu)} \left( \sigma_p - \epsilon \right) + \frac{2}{(1-2\nu)} \frac{(1+\nu)\epsilon}{3(1-\nu)} \left( 1 + \frac{1}{2}\epsilon \right); \]

\[ \sigma_\varphi = \frac{\sigma_p}{(1-\nu)(1-2\nu)} - \sigma_\varphi \text{ sign}(u); \]

\[ \epsilon = \frac{2}{3} \left( \sigma_\varphi - \sigma_p \right) \text{ sign}(u); \]

\[ \omega = \frac{E_\epsilon (1+\nu) - E (1+\nu)}{E (1+\nu)} \frac{\epsilon^{(p-1)-\epsilon}}{\epsilon^i}, \]  
(6)

where \( E_\epsilon \) and \( \nu \) is the elasticity modulus and Poisson ratio in the second linear region of the stress-strain diagram; \( \sigma_p \) and \( \sigma_\varphi \) are radial and tangential strains; \( u = 2(1-\nu)u^*/\alpha T_C r_0 (1+\nu) \) is a dimensionless radial displacement; and \( u \) is the radial displacement.

The Cauchy relations have the form

\[ \epsilon_p = \frac{u}{p}; \epsilon_\varphi = \frac{du}{dp}; e = \frac{1}{3} \left( \frac{du}{dp} + 2 \frac{u}{p} \right). \]  
(7)

These are the fundamental relations used in the following discussion to find the stress-strain state of a hollow sphere whose material possesses elastoplastic properties.

Using the expression for strain and stress intensity (6), the Hooke law for spherical tensors

\[ \sigma = \frac{(1+\nu)}{(1-\nu)(1-2\nu)^2} \epsilon - \frac{2}{(1-2\nu)^2} \theta, \]  
(8)

can be written as follows:

\[ \sigma_\varphi = \frac{1}{3} \sigma_i \text{ sign}(u) = \frac{(1+\nu)}{(1-\nu)(1-2\nu)^2} \left[ \frac{u}{p} - \frac{1}{2} \epsilon_i \text{ sign}(u) \right] - \frac{2}{(1-2\nu)^2} \theta. \]  
(9)

After substituting into it the stress intensities \( \sigma_i \) for the radial stress \( \sigma_p \), the starting equilibrium equation (3) assumes the form

\[ \frac{d\sigma_p}{dp} \left( \frac{d\sigma_p}{dp} + \frac{2}{p} \sigma_i \text{ sign}(u) \right) = 0. \]  
(10)