VIBRATION AND STABILITY OF INTERFERING PLATES IN A FLUID FLOW

V. N. Buivol and Yu. R. Shevchuk

UDC 533.601

The vibration and stability of rectangular (and other shapes) plates in a fluid flow have been investigated with the aid of direct or variational methods [1, 2]. Attempts to apply the second Lyapunov method and the finite element method also proved successful in this case, although these techniques failed to attain widespread usage [3, 4]. Most of the studies in this field, however, are concerned with the behavior of a single plate in an unbounded fluid [5, 6] or in a channel [7, 8, 9, 10]; packets of plates have also been investigated in the form of beams [11]. It is interesting to study the vibration of plate systems in a fluid flow in view of the special operating conditions and vibrational distortions of these plates. Such an analysis makes it possible to determine the critical flow conditions, reveal the effect of one plate on the other, and explain certain experimentally observed vibrational properties of plate systems.

We shall examine the vibration and stability of two parallel plates in an inviscid fluid flow with a free-flow velocity \( V_0 \) (Fig. 1). A potential flow is present, and the thin elastic plates are secured to the walls of the channel.

We will introduce two systems of relative Cartesian coordinates

\[
\xi_j = \frac{x_j}{b}; \quad \eta_j = \frac{y_j}{b}; \quad \zeta_j = \frac{z_j}{b} \quad (j = 1, 2),
\]

each of which is associated with an elastic plate of width \( b \) and rigidity \( D_1 \) and \( D_2 \). These two coordinate systems are connected by the bond

\[
\xi_j = \eta_j; \quad \eta_j = \eta_j; \quad \zeta_j = \zeta_j - \gamma_j, \quad \gamma_j = \frac{a}{b},
\]

where \( a \) denotes the distance between plates.

The equations of motion for the plates take on the following form:

\[
\frac{\partial^4 w_j}{\partial \xi^4_j} + 2 \frac{\partial^4 w_j}{\partial \xi^2 \partial \eta^2_j} + \frac{\partial^4 w_j}{\partial \eta^4_j} + \frac{b^4 \varphi_j}{D_j} \frac{\partial^2 w_j}{\partial \xi^2} = - \left( \frac{1}{\rho_j} \right) \frac{P_j}{D_j} \quad \mid_{\xi_j = 0}.
\]

The pressure \( P_j \) of the fluid on the plate is expressed by the formula

\[
P_j = -p_0 \left( \frac{\partial \varphi}{\partial \eta} + \frac{V_0}{b} \frac{\partial \varphi}{\partial \xi_j} \right),
\]

and the potential \( \varphi \) satisfies the equation

\[
(1 - M_0^2) \frac{\partial^4 \varphi}{\partial \xi^4} + \frac{\partial^2 \varphi}{\partial \eta^2} + \frac{\partial^2 \varphi}{\partial \xi^2} = \frac{b^4}{c_0^2} \frac{\partial^2 \varphi}{\partial \xi^2} + \frac{2M_0 b}{c_0} \frac{\partial^2 \varphi}{\partial \eta \partial \xi},
\]

where \( M_0 = V_0/c_0^{-1} \) is the Mach number of the incident flow, and \( c_0 \) is the speed of sound in an unperturbed fluid.

The boundary conditions along the plate edges will be arbitrary, while the boundary conditions for the potential will be written as follows:

We are searching for the potential $\varphi$ in the class of functions which define the waves propagating along the plates. If $\omega$ is the angular frequency of vibration and $k$ denotes the wave number, then

$$\varphi = \psi(\eta, \xi) e^{i(\omega t - kb)}.$$  

(4)

Given an amplitude $\psi$ for the velocity potential in (2) we obtain

$$\frac{\partial^2 \psi}{\partial \eta^2} + \beta^2 \psi = 0,$$

where

$$\beta^2 = \beta^2 \left[ \left( \frac{V_0 - k_v}{c_0} \right)^2 - 1 \right].$$  

(5)

The function $\psi$ is expressed as the sum of two components, each of which represents a solution to Eq. (5) in the corresponding coordinate system:

$$\psi = \psi_1 (\eta, \xi_1) + \psi_2 (\eta, \xi_2).$$

Furthermore, we determine the function $\psi_1$ on the assumption that there is no second elastic plate present (applying the infinite radiation condition).

Then, taking into account the first condition in (3), we obtain the following for $\psi_1$ and $\psi_2$:

$$\psi_1 = \sum_{n=0}^{\infty} C_n e^{\kappa_n \eta_1} \cos n\pi \eta, \text{where } \kappa_n = \sqrt{n^2 \pi^2 - \beta^2}.$$  

The unknown constants $C_n$ and $D_n$ should be found using the impermeability conditions for plates (3), which also determine the function $w_j$ in terms of the quantities $t$ and $\eta$:

$$w_j = f^{(1)}(\eta) e^{ib(t - \omega t)}. $$

(6)

We obtain the following functional equations for the constants $C_n$ and $D_n$:

$$\sum_{n=0}^{\infty} C_n \cos n\pi \eta - D_n \kappa_n \cos n\pi \eta = ib (\omega - k\nu) f^{(1)}(\eta),$$

(7)

The system of functions $\{ \cos n\pi \eta \}$ ($n = 0, 1, 2, \ldots$) is orthogonal and entire within the interval (0, 1), therefore relations (7) can be expanded in the Fourier series for the functions $f^{(1)}(\eta)$ and $f^{(2)}(\eta)$. Accordingly, the quantities $\kappa_n (C_n - D_n e^{-\kappa_n \eta})$ are the coefficients of the Fourier series expressed as follows

$$\begin{align*}
C_n - D_n e^{-\kappa_n \eta} &= 2ib (\omega - k\nu) \frac{1}{\kappa_n} \int f^{(1)}(\eta) \cos n\pi \eta d\eta, \\
C_n e^{-\kappa_n \eta_1} - D_n &= \frac{2ib (\omega - k\nu)}{\kappa_n} \frac{1}{\delta} f^{(2)}(\eta) \cos n\pi \eta d\eta,
\end{align*}$$

311