Identification of wind stress on shallow water surfaces by optimal smoothing

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Abstract: Using the state space approach, an on-line filter procedure for combined wind stress identification and tidal flow forecasting is developed. The stochastic dynamic approach is based on the linear two-dimensional shallow water equations. Using a finite difference scheme, a system representation of the model is obtained. To account for uncertainties, the system is embedded into a stochastic environment. By employing a Kalman filter, the on-line measurements of the water-level available can be used to identify and predict the shallow water flow. Because it takes a certain time before a fluctuation in the wind stress can be noticed in the water-level measurements, an optimal fixed-lag smoother is used to identify the stress.

Key words: Kalman filtering, Optimal smoothing, Shallow water equations, Wind stress, On-line prediction.

1 Introduction

By representing shallow water flow models in state space form, we are able to use Kalman filtering techniques to identify the shallow water flow. The stochastic-dynamic approach is based on a detailed deterministic model describing the water movement. In order to employ Kalman filtering for the identification and prediction of the shallow water flow using on-line measurements of the water-level, the deterministic model is embedded into stochastic environment by introducing a system noise process. In this way it is possible to take into account the inaccuracies of the underlying deterministic model. By using Kalman filtering, the information provided by the stochastic-dynamic model and the noisy measurements taken from the actual system can be combined to obtain an optimal (least squares) estimate of the system.

In rivers and estuaries a one-dimensional representation of the water movement is quite adequate. In this case the Kalman filter procedure can be based on the De St. Venant equations describing the long wave motion in an open channel (Budgell and Unny, 1980; Brummelhuis et al., 1986, 1988). The resulting filter is applicable to a wide range of practical shallow water flow problems. In the Netherlands a number of stochastic-dynamic tidal models with Kalman filtering procedures have been implemented, e.g. of the Eastern Scheldt estuary, the Western Scheldt estuary and the Rhine-Scheldt channel.

For many applications a one-dimensional representation of the tidal dynamics is not adequate. Although the extension of these one dimensional filtering techniques to two space dimensions does not give rise to conceptual problems, it would impose an
unacceptably greater computational burden. In order to obtain a computationally efficient filter, simplifications have to be introduced. Unfortunately, there are serious problems with the more obvious simplifications one may consider (Millar, 1986; Heemink, 1988a). As a consequence Kalman filtering techniques have yet seldom been applied to realistic two-dimensional filtering problems. Parrish and Cohn (1985) developed a filter for a linear two-dimensional numerical model that is based on the assumption that errors at large distance points are not correlated. As a result the computational complexity of the filter can be reduced dramatically and they showed that their filter is indeed computational feasible for numerical weather prediction. However for most tidal flow problems the domain of the problem is relatively small, so that errors are highly correlated in space and this approach cannot be employed. Therefore, Heemink (1986, 1988a) developed a time-invariant filter approach based on the linear two-dimensional shallow water equations. By using Chandrasekhar-type filter algorithm, the special structure of the filtering problem is exploited to obtain an efficient implementation.

In this paper, building on the ideas presented in Heemink (1988a), we develop an on-line filter procedure for combined wind stress identification and tidal flow forecasting. The approach is based on the linear two-dimensional shallow water equations. In section 2 these equations as well as the boundary conditions are described. By using a finite difference scheme and by defining a state vector that consists of the water-level and velocity at all grid points, the wind stress in some of these grid points as well as some uncertain parameters introduced into the boundary conditions, the model is rewritten in state-space form and embedded into a stochastic environment in section 3. Employing a Kalman filter in section 4, the on-line measurements of the water-level available can be used to estimate the shallow water flow. Because it takes a certain time before a fluctuation in the wind stress can be noticed in the water-level measurements, an optimal fixed-lag smoother is used to identify the space-varying wind stress. This information can be used by the meteorologist to check whether the filter is performing satisfactorily and, if so, to correct the meteorological forecasts. Furthermore, in this way the meteorological input to other models, e.g. for the prediction of wave spectra, can be improved. Both the filter and the smoother are assumed to be time-invariant. As a consequence the time-consuming second moment computations do not have to be recomputed as new measurements become available, but can be solved once and off-line. Furthermore, in this case the fact that the system noise is less spatially variable than the underlying process can be exploited to reduce the computational burden to solve the second moment equations by employing a Chandrasekhar-type algorithm. The number of computations that have to be performed on-line to produce the estimates of the shallow water flow and the wind stress is very limited. In section 5 a number of applications of the approach are described in detail to illustrate the performance of both the filter and the smoother.

2 Shallow water equations

The two-dimensional linear model to describe the shallow water flow in coastal waters consists of the momentum equations:

\[
\frac{\partial u}{\partial t} + g \frac{\partial h}{\partial x} - f v + \frac{\tau_x}{D} - \frac{1}{\rho_w} \frac{\partial p_a}{\partial x} = 0
\]  

(1)

\[
\frac{\partial v}{\partial t} + g \frac{\partial h}{\partial y} + f u + \frac{\tau_y}{D} - \frac{1}{\rho_w} \frac{\partial p_a}{\partial y} = 0
\]  

(2)

and the continuity equation: