DETERMINATION OF STRESS AT THE CONTACT OF A SOIL FOUNDATION WITH A NONFLAT BASE OF THE STRUCTURE

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To estimate the stability of the soil mass serving as the foundation of a concrete structure, the need arises to determine the contact stresses, i.e., the stresses in the soil skeleton at the contact of the base of the structure with the foundation.

In the case of a flat base of the structure, the formula of eccentric compression based on the hypothesis of the linear distribution of stresses over the calculated section is widely used for determining normal stresses on the contact plane. In the case of a nonflat base, this hypothesis can be generalized, having applied it to the total stress in the soil skeleton with normal and shear stresses. In this case, the total stress at any point is taken proportional to the movement of this point, which in turn is proportional, under conditions of the two-dimensional problem, to the distance between the point of the base at which the stress is determined and a fixed axis about which the structure turns during differential settlement [1].

It should be noted that this approximate method based on the hypothesis of a generalized "modulus of subgrade reaction" does not take into account deformations of the foundation beyond the limits of the base of the structure. However, to solve the problem of stability of earth masses according to the scheme of the first group of limit states with the use of the Coulomb–Mohr soil strength criterion, this circumstance, we must assume, does not strongly affect the calculation result. Therefore, the aforementioned method is recommended for use by the building code SNiP [2] and guide [3].

The hypothesis about a direct proportionality between the total stress in the soil skeleton at any point of the contact surface and its movement can be considered acceptable for performing engineering calculations so long as the stresses in the foundation soil do not reach the ultimate strength. The aforementioned direct proportionality is disturbed on some part of the contact surface when this strength is reached, and the shear stress on this part should be determined by the Coulomb relation

\[ \tau = \tau_u = \sigma + c, \]

where \( \sigma \) is the normal stress on the investigated part of the contact surface; \( f, c \) are soil strength parameters.

Ignoring this circumstance leads to considerable calculation errors. The derivation of formulas is given below for determining stresses of the contact surface with consideration of the soil reaching the ultimate strength on some part of the contact surface.

Let us examine the general statement of the problem of determining the reaction of a soil foundation under conditions of plane strain, i.e., under the assumption that the foundation of a sufficiently extended structure is loaded uniformly over the length. Under these conditions, the surface of the base of the structure contacting the foundation can be represented by its profile – by a curve or broken line – as well as by their combination (Fig. 1). Relative to the soil foundation, the concrete structure can be considered a perfectly rigid, nondeformable body.

It is assumed also that the shear surfaces on which the shear stress in the soil reached its limit coincide only with certain parts of the contact surface. Other shear surfaces do not exist in the foundation, i.e., the soil mass of the foundation has a margin of stability. The foundation soil on different parts of the profile of the base can have different values of the strength parameters \( f \) and \( c \), but the same yielding.

The condition of nondeformability of the base of the structure makes it possible to represent the movement \( \eta \) of any point of the profile of the base in the form of a certain sum of the forward movement of the profile \( \eta_0 \) directed at a certain angle \( \varepsilon \) to the vertical and of the movement \( \eta_w \) of the investigated point caused by turning of the profile relative to an arbitrarily selected pole 0 through a certain angle \( \omega \) (Fig. 1). The latter movement, with an accuracy to second-order infinitesimals, is proportional to the distance \( r \) from the selected pole 0 to the point of the profile of the base at which the movement \( \eta_w \) is determined.
Fig. 1. Calculation scheme of movements of the non-deformable profile of the base of the structure. The line AB indicates the position of the profile before movement; A'B' corresponds only to forward movement; A'B'' corresponds to the total movement of the profile.

We will place the origin of a rectangular coordinate system with horizontal axis X and vertical axis Z at the arbitrarily selected pole 0. In this coordinate system the components of the total movement of any point of the base of the structure in the normal (η_0) and tangential (η_1) directions to the profile are determined with an accuracy to second-order infinitesimals by the relations:

\[ η_0 = η_0 \cos (α + ε) + o \cos (α - β); \]
\[ η_1 = η_0 \sin (α + ε) + o \sin (α - β), \]

where \( α \) is the angle of slope of the profile of the base at the investigated point to the X axis; \( β \) is the angle between the X axis and direction formed by radius r. The positive direction of reading the angles \( α \) and \( β \) is toward the shortest turn from the X axis to the Z axis and for angle \( ε \) it is in the opposite direction.

The stresses in the soil skeleton at the contact with the base of the structure are determined by the relations:

\[ \sigma = k_p [η_0 \cos (α + ε) + o \cos (α - β)], \]
\[ τ = k_p [η_0 \sin (α + ε) + o \sin (α - β)], \]
\[ 0 < \tau < τ_{11}, \]
\[ τ = τ_{11}, \]

where \( k_p \) is the factor of proportionality between stresses and movements — the generalized modulus of subgrade reaction.

In the adopted coordinate system the equations of equilibrium of the forces acting on the structure and their moments have the form

\[ \sum X = 0: \int_A^{B} (τ \cos α - a \sin α)ds = N \sin δ, \]
\[ \sum Z = 0: \int_A^{B} (τ \sin α + a \cos α)ds = N \cos δ, \]
\[ \sum M_0 = 0: \int_A^{B} r [τ \sin (α - β) + a \cos (α - β)]ds = N e_0, \]

where \( N \) is the resultant of the active forces acting on the structures; \( δ \) is its angle of slope to the vertical axis, the positive direction of reading of which is realized by the shortest turn from the Z axis to the X axis; \( e_0 \) is the eccentricity of this force relative to pole 0; \( ds \) is an element of the profile of the base.