As we know, saltation is the most common form of sediment movement in the near-bottom region of a channel flow [10].

The saltation of sediment contributes to its collision with particles projecting from the bottom, and also to Magnus' force [2]. This makes it possible to assign initial conditions for the separation of particles from the bottom of the flow in the form of characteristic flow velocities and their dispersion, while certain new experimental data [13] enable us to parameterize the path of the saltating sediment, which can be described by an equation of particle motion in a liquid flow that is more general than the previous one [10, 13].

This equation assumes the form

$$ A_1 d^3 (\rho_d d\vec{v}/dt + \alpha \rho (d\vec{v}/dt - d\vec{v}/dt)) = \vec{F}_d + \vec{F}_g + \vec{F}_p + \vec{F}_t $$

(1)

where $\vec{F}_d = \rho c_2 A_2 d^2 (\vec{v} - \vec{v}_4) (\vec{v} - \vec{v}_4) / 2$ is the vector of the force of a particle's hydrodynamic resistance, $\vec{F}_g = (\rho_0 - \rho) g A_3 d^3$ is the vector of the gravitational force after deduction of Archimedes' force, $\vec{F}_p$ is the force vector associated with the pressure gradient under accelerated motion of the liquid, $\vec{F}_t$ is the vector of Bass' force, which takes into account the effect of the deviation of the liquid flow from a stationary state, $A_1$ and $A_2$ are particle-shape factors with respect to the volume and area of a characteristic section, respectively [5], $d$ is the average particle size [4, 5], $\alpha$ is the coefficient of the associated mass, $g$ is the vector of the acceleration of gravity, $c$ is the drag of the particle as it flows through the liquid, and $\vec{v}$ and $\vec{v}_4$ are velocity vectors of the liquid flow and particle motion, respectively.

Let us introduce the following assumptions:

the flow of liquid is considered stationary and uniform, and the forces $\vec{F}_p$ and $\vec{F}_t$ can be neglected in Eq. (1);

the coefficient of the associated mass is constant during a particle jump, but varies near the bottom;

for the nonsteady motion of a particle, its drag also depends on the relative displacement rate, just as in the case of steady motion. For vertical particle displacement, it is then assumed that

$$ F_g = \rho c_2 A_2 d^2 (w - w_4) (w - w_4) / 2, $$

(2)

where $c_2$ is the drag of the particle as it flow through the liquid in the vertical direction, and $w$ and $w_4$ are the vertical velocity components of the flow and the partial displacements, respectively; and, the vertical velocity component of the flow assumes a constant positive value during the particle-jump period, since the dimensions and duration of the particle jumps [10, 13] are small as compared with the scales of turbulence and the averaging period of the measured flow characteristics [11, 13] in connection with the fact that the vertical velocity component of the flow, which is determined during the period of the particle jump may assume values other than zero; separation of particles from the bottom is observed only then and there where an increase develops in the velocity pulsations of the flow and the pulsations of the vertical velocity component of the flow are directed upward in the vicinity of the separating particle [10].

The assumption that we have made makes it possible to represent the paths of the saltating particles as smooth curves. In this case, the value of the vertical velocity component of the flow, which is averaged over the period of the jump, will be considered proportional to the pulsation standard of this component [7].

These four assumptions make it possible to rewrite Eq. (1) as

$$ A_1 d^3 (\rho_d - \rho) d\vec{w}/dt = \rho c_2 A_2 d^2 (w - w_4) / 2 - (\rho_0 - \rho) g A_3 d^3. $$

(3)
The solution of Eq. (3) will be sought for the following initial conditions:

\[ \omega|_{t=0} = \omega_0; \]

\[ z|_{t=0} = 0. \]

The force of the particle's resistance to displacement will be positive or negative depending on the sign of the difference between the velocities of the flow and particle. Two cases should therefore be examined.

1. \( w \geq w_{49} \). Let us set \( \dot{w} = w - w_{49} \); Eq. (3) will then be rewritten as

\[ \frac{d\dot{w}}{dt} = -\Omega \dot{w}^3 + \theta, \]

where

\[ \Omega = \rho c_A \] / \[2A_1 d(\rho_1 + \rho_2)\];

\[ \theta = \frac{g(\rho_1 + \rho)}{\rho_1 + \alpha \rho}; \]

and \( \theta_1 = \sqrt{\Omega} \) is the fall diameter of the particle.

Equation (6) has the solution [3].

\[ -\Omega t = \ln \left( \frac{\dot{w} - \omega}{\dot{w} + \omega} \right) / (2\omega) + \epsilon. \]

If the particle moves with a negative acceleration, the condition \( \dot{w} > \omega \) should be fulfilled from Eq. (6); this is obvious in general.

Having determined the constant of integration in Eq. (9) from condition (4), we obtain

\[ -\Omega t = \ln \Phi \omega - \dot{w} / (\omega + \dot{w}), \]

where

\[ \Phi = \omega + w - w_{49} \]

\[ \omega - w + w_{49}. \]

From Eq. (10) we find

\[ \dot{w} = \omega - \frac{2\omega}{\Phi(\exp 2\omega dt) + 1}, \]

or

\[ dz/dt = \dot{w} - \omega + \frac{2\omega}{\Phi(\exp 2\omega dt) + 1}. \]

Equation (12) with initial condition (5) has the solution

\[ z = (\dot{w} - \omega)t + 2\omega \int_0^t \frac{dt}{\Phi(\exp 2\omega dt) + 1}. \]

Time \( t_{\text{max}} \) at which the particle attains the maximum height \( z_{\text{max}} \) and the height of the jump itself will be, respectively,

\[ t_{\text{max}} = - \ln \Phi \omega - \dot{w} / (\omega + \dot{w}) / (2\omega); \]

and

\[ \frac{1 + \Phi}{(\exp -2\omega dt_{\text{max}} + \Phi)} \]

\[ / \Omega. \]