APPORXIMATE CALCULATIONS OF LOCAL RESISTANCE COEFFICIENTS IN THE CASE OF A SUPERCRITICAL FLOW REGIME

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Three types of flows, to which correspond various profiles of the water surface, can be observed on a stretch of local resistance. Subcritical flow is the most common case and the corresponding procedures of calculation given in the literature pertain precisely to this type of flow [1]. However, in a study of erosion processes (overland runoff) one often encounters a supercritical flow, for which the water level in a contraction will rise, the water surface is uneven, and waves can be observed on the upstream and downstream stretches of the flow (Fig. 1).

Let us examine a steady free supercritical fluid flow in a canal with width $b > > h$, bottom slope angle $\beta$, and depth $h$. During flow of a viscous fluid its total specific energy (potential and kinetic) decreases by virtue of various hydraulic resistances. The corresponding energy equation has the form

$$a_1 + h_1 + \alpha v_1^2 / (2g) = a_2 + h_2 + \alpha v_2^2 / (2g) + h_{1-2},$$

where $a$ is the bottom elevation, $m$; $h$ is the depth of the flow, $m$; $v$ is the average flow velocity, m/sec; $\alpha$ is the kinetic-energy correction factor; $g$ is the acceleration of gravity, m$^2$/sec; $h_{1-2}$ is the losses of specific energy on overcoming friction forces, $m$; 1, 2 are indices indicating to which sections the quantities pertain.

In the case of a local resistance (grate) between the calculated sections the equation will take the form

$$a_i + h_i + \alpha v_i^2 / (2g) = a_{i+1} + h_{i+1} + \alpha v_{i+1}^2 / (2g) + h_{s,i} + h_{i+1},$$

where $h_s$ is losses of specific energy as a consequence of the effect of local resistance, $m$; $s$ is an index signifying that the indicated quantities pertain to the calculated section with a local resistance in front of it.

Since the zone of influence of the local resistance does not extend to calculated section 1-1 (Fig. 1), the left sides of Eqs. (1) and (2) are equal. Equating the right sides of these equations, we can determine $h_s$:

$$h_s = h_2 - h_1 + \frac{\alpha}{2g} \left( v_2^2 - v_1^2 \right).$$

In the case of supercritical flow the determination of the quantities figuring in the equation is characterized by low accuracy. In particular, with insignificant volume flow rates the accuracy of measuring the depth is estimated with a considerable relative error. This is related to one of the most important properties of turbulence — disorder of motion.

Having expressed the depth $h$ by the ratio of the unit discharge of water $q$ to the average velocity $v$, we obtain

$$h_s = q \left( 1 - 1 / v_s \right) + \frac{\alpha}{2g} \left( v_2^2 - v_1^2 \right);$$

$$q = Q / b = vh,$$

where $q$ is the discharge of water per unit width of the flow, the unit discharge, m$^2$/sec; $Q$ is the volume flow rate of water, m$^3$/sec; $b$ is the width of the flow, m.
The velocity of water in section 2-2 can be determined on the basis of the range of a free hydraulic jet (Fig. 2). Under real conditions the range of a hydraulic jet is affected by air resistance and complex oscillatory phenomena occurring in the jet, disturbing its compactness [2]. According to the conditions of conducting the experiment, the fall of the free hydraulic jet occurs from a certain height at a negative angle to the line of the horizon, as a consequence of which the effect of the air resistance forces is brief for a particular point of the jet, and therefore the latter can be neglected.

To derive the equation of the flight path of the free hydraulic jet, we introduce in the vertical plane a local coordinate system, and having matched its origin with the initial point of the path, we direct axis $x_2$ horizontally and axis $y_2$ vertically downward (Fig. 2). Then the flight path of the jet can be described by the following equations:

$$x_2 = v_2 \cdot \cos \beta \cdot t;$$

$$y_2 = v_2 \cdot \sin \beta \cdot t + \frac{g t^2}{2},$$

where $t$ is time, sec, $v_2$ is the average velocity of the jet at the initial point of the path, m/sec; $\beta$ is the bottom slope angle, deg.

Expressing $t$ from Eq. (6) and substituting into (7), we obtain

$$y_2 = x_2 \cdot \tan \beta + \frac{g x_2^2}{2v_2^2 \cos^2 \beta},$$

The height from which the hydraulic jet falls is $y_2 = a_2$, since according to the conditions of conducting the experiment $a_2 \gg h_2$. Then the average flow velocity in section 2-2 can be determined by the formula