Supersymmetry and Random Sources

Pinaki Roy
Electronics Unit, Indian Statistical Institute, Calcutta—700035, India

Received 3 October 1983

Abstract. We evaluate the one loop effective potential for the supersymmetric Higgs model in the presence of a gaussian random source and examine supersymmetry breaking at the one loop level.

1. Introduction

In recent years, there have been much interest in quantum field theories in the presence of background fields which are stochastic in nature [1–3]. The results of these investigations have raised possibilities of application of random source in the description of QCD quark confinement [4–5]. Also there have been stochastic approach to supersymmetry [6]. It is likely that applications would be found in grand unified theories. In the present article we shall consider the super Higgs model in the presence of a random source and examine if it induces supersymmetry breaking at the one loop level. The plan of the paper is as follows: in Sect. 2 we describe the model and the formalism of constructing the effective potential; in Sect. 3 we study the minima of the effective potential and comment on supersymmetry breaking and Sect. 4 is devoted to conclusion.

2. Super Higgs Model and Evaluation of the One Loop Effective Potential

The supersymmetric Higgs model is described by the following Lagrangian [7].

\[ \mathcal{L} = - \frac{1}{4} F_{\mu \nu}^2 + \frac{1}{2} \bar{\lambda} \gamma^\mu \lambda \gamma^\nu \psi \gamma^\lambda \psi + \left( D_\mu \psi \right)^* \left( D^\mu \psi \right) - ie \sqrt{2} \left( \bar{\psi} \gamma^\lambda \lambda \psi - \phi^* \gamma^\lambda \lambda \psi \right) + \frac{1}{2} \left( \partial^\lambda A_\lambda \right)^2 \]

where \( F_{\mu \nu} = \partial^\mu A_\nu - \partial^\nu A_\mu \) and \( D_\mu \psi = D_\mu \psi - ie A_\mu \psi \).

Here \( \psi \) is a charged scalar, \( \psi \) a left handed Dirac spinor, \( \lambda \) a Majorana spinor and \( A_\mu \) the gauge boson.

This model, in addition to being invariant under the global supersymmetry transformation, is also invariant under the following local gauge transformation (the gauge group being \( U(1) \)):

\[ \begin{align*}
\delta A_\mu &= \frac{1}{e} \partial_\mu \xi(x) \\
\delta \lambda &= 0 \\
\delta \phi &= -i \phi \xi(x) \\
\delta \psi \lambda &= -i \psi \xi(x)
\end{align*} \]

The scalar potential is given by

\[ V(\phi) = \frac{1}{2} \left( \phi^* + e \phi^* \xi \phi \right) \]

Now two situations may arise, depending on the sign of the parity violating parameter \( \xi \):

Case 1: \( \xi < 0 \). In this case the minimum of the potential occurs when \( \left< |\phi|^2 \right> = -\xi/e \) and the \( U(1) \) gauge symmetry is broken while supersymmetry remains unbroken.

Case 2: \( \xi > 0 \). In this case supersymmetry is broken spontaneously at the tree level and gauge symmetry remains unbroken.

We now consider the system in the presence of a gaussian random source \( h \). The probability distribution of this source is given by [8].

\[ P[h] = \exp \left\{ - \frac{1}{2\Delta} \int h^2(x) dx \right\} \]

where \( \Delta = 4\pi\mu^2 \beta(h) \) with \( \beta(h=0)=0 \) and \( \mu \) has the dimension of mass.

To calculate the effective potential, the external source \( h \) are coupled to the scalar fields. The averaged connected generating functional (the averaging is carried out w.r.t. \( P[h] \)) is then given by

\[ W[\phi] = \int dh P[h] \ln z[\phi + h] \]

where

\[ z[\phi + h] = \int [d\phi] e^{i \phi^* \xi \phi + \phi h} \]
(dA) standing for all the fields in the theory (in (2.5), we have suppressed the fermionic source terms).

To perform the integration in (2.5) we use the following technique: [9]:

\[ \ln z = \lim_{n \to 0} \frac{z^n - 1}{n} \quad \text{(2.7)} \]

Then (2.5) gives

\[ \mathcal{W}[\phi] = \lim_{n \to 0} \frac{1}{n} \left[ d\phi_i \right] \cdot \exp \left[ - \int d^4x \left( L_{\text{eff}} - j \sum_{i=1}^n \phi_i + \text{other terms} \right) \right] \quad \text{(2.8)} \]

Here,

\[ L_{\text{eff}} = \frac{1}{2} \phi_i \left( \left( -\square + e^2 \xi \right) \delta_{ij} - \Delta \Pi_{ij} \right) \psi_j + \text{other terms} \]

(2.9)

where \( \Pi_{ij} = 1 \) for all \( i \) and \( j \) and \( \Pi^2 = n \Pi \). We note that as a result of the last relation, in the limit \( n \to 0 \), the random source induced part of the propagator occurs only in closed loops and not on external lines.

Now making the shift,

\[ \phi_i \to \phi_i' \]

we find after a straightforward calculation

\[ V_1 = -\frac{i}{2} \int \frac{d^4k}{(2\pi)^4} \left[ -3 \ln(-k^2 + M_2^2) + \ln(-k^2 + R_1^2) + \ln(-k^2 + M_4^2) \right] - \frac{A^2}{64\pi^2} \ln \frac{M_2^2}{A^2} \]

(2.11)

where

\[ M_1^2 = e^\xi + e^2 \phi^2 - \Delta + \frac{e^4 \phi^4}{4} \]

(2.12)

\[ M_2^2 = e + e^2 \phi^2 - \Delta - \frac{e^4 \phi^4}{4} \]

(2.13)

\[ M_\psi^2 = e^2 \phi^2 \]

(2.14)

\[ M_\gamma^2 = 2e^2 \phi^2 \]

(2.15)

and \( R_1^2 \) and \( R_2^2 \) are the roots of the equation

\[ x^2 - M_2^2 x + \alpha M_2^2 M_\psi^2 = 0 \]

(2.16)

(It is to be noted that for \( \Delta = 0 \), the result agrees with the one without random source; [10]):

Choosing Landau gauge and using dimensional regularization, we obtain from (2.11)

\[ V_1 = -\frac{1}{64\pi^2} \left[ (M_1^2 - \Delta^2) \ln \frac{M_1^2}{A^2} + (M_2^2 - \Delta^2) \ln \frac{M_2^2}{A^2} \right. \]

\[ + \left. 3 M_\psi^4 \ln \frac{M_\psi^2}{A^2} - 4 M_\gamma^4 \ln \frac{M_\gamma^2}{A^2} \right] \]

(2.17)

\( A^2 \) being an arbitrary renormalization scale.

Note: \( \pi \) stands for \( \pi = \frac{4}{3} \).

### 3. Analysis of the Effective Potential

The complete one loop potential is given by

\[ V = V_0 + V_1 = \frac{i}{2} \left( \xi + e \phi^2 \right) \]

\[ + \frac{1}{64\pi^2} \left[ (M_1^4 - \Delta^2) \ln \frac{M_1^2}{A^2} + (M_2^4 - \Delta^2) \ln \frac{M_2^2}{A^2} \right. \]

\[ + \left. 3 M_\psi^4 \ln \frac{M_\psi^2}{A^2} - 4 M_\gamma^4 \ln \frac{M_\gamma^2}{A^2} \right] \]

(3.1)

Now there could be two alternatives: We may start with (1) spontaneously broken gauge symmetry (2) spontaneously broken supersymmetry.

**Case 1.** In this case we have \( \xi < 0 \) and the minimum of the tree level potential occurs at \( \phi^2 = -\xi/e \). If supersymmetry is to be broken spontaneously by the one loop correction, then the broken gauge symmetry has to be restored i.e. the minimum should be shifted from \( \phi^2 = -\xi/e \) to \( \phi^2 = 0 \). From (3.1) we find that \( \phi = 0 \) can not be solution of the equation

\[ \frac{\partial V}{\partial \phi^2} = 0 \]

(3.2)

and therefore can not be a minimum of the expression (3.1), no matter whatever be the value of \( \Delta \). However for \( \Delta = 0 \), we find that

\[ V_1 = 0 \]

(3.3)

and this expresses the fact that radiative correction do not break supersymmetry.

**Case 2.** In this case, we have \( \xi > 0 \). To analyse this case, we write \( V \) in the following way:

\[ V = V_0 + V_1 + V_2 \]

(3.4)

where

\[ V_0 = \frac{i}{2} \left( \xi + e \phi^2 \right)^2 \]

\[ V_1 = \frac{1}{64\pi^2} \left[ M_1^4 \ln \frac{M_1^2}{A^2} + M_2^4 \ln \frac{M_2^2}{A^2} \right. \]

\[ + \left. 3 M_\psi^4 \ln \frac{M_\psi^2}{A^2} - 4 M_\gamma^4 \ln \frac{M_\gamma^2}{A^2} \right] \]

\[ V_2 = -\frac{A^2}{64\pi^2} \left[ \ln \frac{M_1^2}{A^2} + \ln \frac{M_2^2}{A^2} \right] \]

(3.5)

It is seen that for large values of \( \phi \), \( (V_0 + V_1) \) dominates over \( V_2 \); while for small values of \( \phi \), \( V_2 \) dominates over \( (V_0 + V_1) \). Therefore for values of \( \phi \) near the origin, we have

\[ V = -\frac{A^2}{64\pi^2} \left( \ln \frac{M_1^2}{A^2} + \ln \frac{M_2^2}{A^2} \right) \]

(3.6)

To obtain a qualitative result, we choose \( \Delta = e^\xi \). Then the minimum is given by

\[ \phi_{\text{min}}^2 = \frac{4(e^\xi - \Delta)}{3e^2} \]

(3.6)