The analogy between fluid friction and heat transfer of laminar forced convection on a flat plate

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Abstract. This paper has examined the validity of the analogies between heat transfer and fluid friction when they are applied to laminar forced convection on a flat plate. For the case of uniform wall temperature, all the analogies are valid for Prandtl number do not differ greatly from 1, and the Colburn analogy is consistent with numerical data for Pr ≥ 1. However, the previous analogies are not valid for the case of uniform wall heat flux. For fluids of Pr ≥ 1 under uniform heating, the Colburn analogy factor is 0.72. In addition, these analogies are not applicable to the fluids of small Prandtl number. For the sake of completeness, this work has developed the analogies between heat and momentum transfer over the range of 0.001 ≤ Pr ≤ 0.1 for both the cases of constant wall temperature and heat flux.

Nomenclature

c constant
C_f local friction coefficient, 2 μ/ρ u^2
C_f average friction coefficient, 2 μ/ρ u^2
j Colburn j factor, Nu/Re Pr1/3
l Colburn l factor, Nu/Re Pr1/3
h local heat transfer coefficient
h average heat transfer coefficient, \frac{1}{x} \int_0^x h dx
k thermal conductivity of fluid
Nu local Nusselt number, h x/k
Nü average Nusselt number, h x/k
Pe Peclet number, Pr Re
Pr Prandtl number, v/α
q_w wall heat flux
Re Reynolds number, u_∞ x/ν
T fluid temperature
T_w wall temperature
T_∞ free stream temperature
u velocity component in x direction
u_∞ velocity of free stream
v velocity component in y direction
x coordinate parallel to the plate
y coordinate normal to the plate

Greek symbols

α thermal diffusivity
θ dimensionless temperature for the UWT case,
(T - T_w)/(T_∞ - T_w)
v kinematic viscosity
ρ density
τ_w wall shear stress, μ (D/∂y)\|y=0
τ_a average wall shear stress, \frac{1}{x} \int_0^x τ_x dx
ϕ dimensionless temperature for the UHF case,
[(T - T_∞)/(q_w x/k)] [(Pr Re)^{1/2}/(1 + Pr)^{1/6}]

Subscripts

w adjacent to the wall
∞ far from the wall

1 Introduction

The analogy that relates heat transfer to wall friction was introduced originally by Reynolds [1] for turbulent flow in pipe. The Reynolds analogy can be derived from an analysis on the turbulent core with some assumptions. Prandtl [2] and Taylor [3] modified, independently, the Reynolds analogy by including the viscous sublayer in their analysis. Von Karman [4] further modified the Prandtl analogy by considering the buffer region in addition to the turbulent core and the viscous sublayer. Although the Prandtl and Karman analogies get some improvements, they are not satisfactory for Prandtl number differ significantly from unity. For fluids of high Prandtl number, the analogy of Colburn [5], which is based on experimental data of both laminar and turbulent flow, has been very successful and most widely used.

The above analogies have been extensively applied to various flows and geometries under the restriction that no
form drag is present, such as flows in pipes and over a flat plate. When any analytical, computational, or experimental results of wall friction is available, the heat transfer Nusselt number or the mass transfer Sherwood number can be estimated readily by using these analogies.

As already mentioned before, all the previous analogies are developed from theoretical analyses or experimental data of turbulent flow in a pipe of uniform wall temperature. Therefore, the validity of these analogies should be examined when they are applied to the laminar forced convection flow on a flat plate. One of the aims of this work is to clarify the applicability of these analogies on laminar flow past a flat plate, especially for the case of uniform heat flux.

It is worth to note that all the previous analogies can not be applied to liquid metals of low Prandtl number (Pr < 0.1), as will be shown in this paper. For this reason, another purpose of this work is to develop the analogies for the laminar flow of low Prandtl-number fluids over a flat plate.

2 Comparison of the analogies for large Prandtl numbers

Very precise similarity solutions of laminar forced convection heat transfer from flat plates with uniform wall temperature (UWT) and uniform wall heat flux (UHF) have been reported [6] for any Prandtl number between 0.0001 and infinity. The local Nusselt number of the UWT case is related to the numerical data of the gradient of the dimensionless temperature at the wall, $\theta'(0)$, by

$$\frac{Nu}{Re^{1/2}} = \frac{Pr^{1/2}}{(1 + Pr)^{1/6}} \left[ 1 - \theta'(0) \right].$$

The numerical results of $Nu$ for $Re = 10000$ over the range of $0.01 \leq Pr \leq 1000$ are compared in Fig. 1 with the previous analogies:

Reynolds analogy:

$$\frac{Nu}{Re} = \frac{C_f}{2},$$

Prandtl analogy:

$$\frac{Nu}{Re Pr} = \frac{C_f}{2} \left[ 1 + 5 \sqrt{\frac{C_f}{2}} (Pr - 1) \right]^{-1},$$

Von Karman analogy:

$$\frac{Nu}{Re Pr} = \frac{C_f}{2} \left[ 1 + 5 \sqrt{\frac{C_f}{2}} \left( Pr - 1 + \ln \left( \frac{5 Pr + 1}{6} \right) \right) \right]^{-1},$$

Colburn analogy:

$$j = \frac{Nu}{Re Pr^{1/3}} = \frac{C_f}{2}.$$  

For laminar flow on a flat plate, the friction factor is well known as [6]

$$\frac{C_f}{2} = 0.33206 Re^{-1/2}.$$  

which has been substituted into Eqs. (2) to (5) for plotting Fig. 1.

For the laminar forced convection flow on an isothermal flat plate, the Nusselt number predicted from the Colburn analogy is fairly close to the numerical data for $Pr \geq 0.1$ as can be seen from Fig. 1. In addition, all the analogies are valid at values of Prandtl number around unity. However, Fig. 1 reveals that the analogies of Reynolds, Prandtl and Karman are not satisfactory for Prandtl number differ greatly from 1.

3 Comparison of Colburn analogy factor

A further examination on the Colburn analogy will be made in this section. For the UWT case, the numerical values of the Colburn analogy factor, $(C_f/2)/(Nu/Re Pr^{1/3})$, can be calculated from a combined equation of Eqs. (1) and (6):

$$\frac{C_f}{2} \frac{Nu}{Re Pr^{1/3}} = \left( \frac{Pr}{1 + Pr} \right)^{1/6} 0.33206 \frac{1}{1 + \theta'(0)}. \tag{7}$$

The numerical results of the Colburn analogy factor calculated from this equation for some specified Prandtl numbers between 0.7 and 10000 are presented in Table 1. It is found that the Colburn analogy factor for a plate with uniform wall temperature is fairly close to unity (0.98 ~ 1.01) for $Pr \geq 0.7$.

Since the average Nusselt number $\overline{Nu} = 2 Nu$ for the case of uniform wall temperature, and the average friction coefficient $\overline{C_f} = 2 C_f$ for laminar flow over a flat plate, therefore

$$\frac{C_f}{2} \frac{Nu}{Re Pr^{1/3}} \approx 0.98 \left( \frac{1}{Nu} \right) \frac{Nu}{Re Pr^{1/3}} \approx 0.98.$$  

![Fig. 1. Comparison of Nu estimated from various analogies with numerical data, Re = 10000](image-url)