Spin-Spin Asymmetries in Inclusive Muon Proton Charm Production

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Abstract. Spin-spin asymmetries have been calculated in the inclusive process \( \mu^+ p \rightarrow \mu c\bar{c}X \) with reference to photon gluon events only. The resulting asymmetries are small and relatively insensitive to the exact form of the structure functions used.

1. Introduction

Recent experimental work [1] has demonstrated that the QCD Bethe Heitler process can be uniquely distinguished in the interaction \( \mu^+ p \rightarrow \mu c\bar{c}X \). Assuming that charmed meson decay generates a clear dimuon or trimuon signature, the data is well represented by the photon gluon model for \( c\bar{c} \) production. This suggests that an experimental technique now exists for probing the gluonic structure of the nucleon, independently of the quark structure. To date, little is known about the gluon structure, and in particular the gluon spin structure, of the nucleon.

Since the full dynamics of quark gluon systems cannot yet be calculated, their analysis pivots on the use of structure functions, distributions in terms of the variable \( \eta \), naively that fraction of the nucleon momentum carried by the parton in question. Although the inclusive distribution functions are controlled by non-perturbative effects, QCD predictions for spin correlations in the region \( \eta \rightarrow 1 \) are possible [2, 3]. From the study of perturbation tree diagrams in QCD, and postulating the gluon component as arising via a bremsstrahlung process \( q \rightarrow q\gamma \) from a valence quark, a hypothetical form for the inclusive gluon helicity structure functions may be written down [3]

\[
\begin{align*}
G_{\gamma^+} (\eta) &= c/\eta (1-\eta)^2 [2 + (1-\eta)^2] \\
G_{\gamma^-} (\eta) &= c/\eta (1-\eta)^2 [1 + 2(1-\eta)^2]
\end{align*}
\] (1)

where \( G_{a\gamma^\pm} (\eta) \) refers to a gluon of helicity \( \pm \) from a nucleon of helicity \( \pm \). By parity \( G_{+\gamma^+} (\eta) = G_{-\gamma^-} (\eta) \) and \( G_{-\gamma^+} (\eta) = G_{+\gamma^-} (\eta) \).

The aims of this work are two fold: firstly, to calculate for \( \mu^+ p \rightarrow \mu c\bar{c}X \) the spin-spin asymmetry resulting from the general ansatz of equation (1) and determine whether it is likely to be experimentally observable and, secondly, to attempt a distinction between different gluon helicity models.

The asymmetry calculated may be written

\[
A = \frac{d\sigma_{++} - d\sigma_{+-}}{d\sigma_{++} + d\sigma_{+-}}
\]

where

\[
d\sigma_{++} = \frac{d\sigma}{dQ^2 d\Omega d\phi} (\mu^+_e + p_+ \rightarrow \mu c\bar{c}X)\text{etc.}
\]

The photon direction defines the positive \( z \) axis, the sense of the azimuthal angle \( \phi \) being a clockwise rotation of the lepton plane about \( z \) when viewed along positive \( z \). The angle \( \phi \) is that between the normals of the lepton and quark planes. Experimentally the quantity \( \phi \) will be hard to measure, since this is revealed by a study of final state jet structure. However, a jet analysis cannot uniquely identify the Bethe Heitler process from the Compton; to identify the Bethe Heitler process, the multimuon signature must be observed, and \( \phi \) determination forfeited.

The usual assumption is made, factorising the cross section into a convolution of a structure function and a hard scattering subprocess \( d\hat{\sigma}_{ab} (\mu_b g_b \rightarrow \mu c\bar{c}) \). Then

\[
\begin{align*}
d\sigma_{++} &= G_{++} d\hat{\sigma}_{++} + G_{+-} d\hat{\sigma}_{+-} \\
d\sigma_{+-} &= G_{+\mp} d\hat{\sigma}_{++} + G_{-\mp} d\hat{\sigma}_{+-}
\end{align*}
\]

where by parity \( d\hat{\sigma}_{+-} = d\hat{\sigma}_{-+}, d\hat{\sigma}_{++} = d\hat{\sigma}_{-\mp} \) which gives the more useful form of the asymmetry as

\[
A = \frac{\Delta G \Delta d\hat{\sigma}}{2G d\hat{\sigma}}
\]

\[
G = (G_{++} + G_{+-}) = \frac{3c}{\eta} (1-\eta)^2 [1 + (1-\eta)^2]
\]

\[
\Delta G = (G_{++} - G_{+-}) = c(1-\eta)^2 (2-\eta)
\]

\[
d\hat{\sigma} = \frac{1}{2} (d\hat{\sigma}_{++} + d\hat{\sigma}_{+-}) \quad \Delta d\hat{\sigma} = d\hat{\sigma}_{++} - d\hat{\sigma}_{+-}
\]
2. Calculation

The calculation is the evaluation of the lowest order QCD Bethe Heitler diagram with an attached lepton tensor (Fig. 1). The gluon is taken as on shell, $q^2 = 0$. It is assumed that an offshell gluon contribution would be suppressed by the propagator [4]. As a high energy approximation, the incoming and outgoing muons are taken to be massless; quark masses are retained.

A convenient form analogous to the simple on shell process can be obtained by writing

$$\varepsilon = (2Q^2)^{-1/2} \frac{v_1 u(1)}{v_2 u(2)}$$

The components of the "virtual photon polarization vector" are evaluated using the lepton tensor [5], and are expressed in terms of a polarization parameter $\varepsilon$, a measure of the polarization state of the photon. Define $\varepsilon$ by

$$\varepsilon \cos 2\phi = \left[ \frac{\varepsilon_x^2}{\varepsilon_y} - \frac{|\varepsilon_x|}{\varepsilon_y} \right] = \frac{L_{xx} - L_{yy}}{L_{xx} + L_{yy}}$$

which becomes

$$\varepsilon^{-1} = 1 + 2 \frac{|k|^2}{Q^2} \tan^2 \theta/2$$

in the $m^2 \to 0$ limit.

The final form of $\varepsilon$ is

$$(1 - \varepsilon)^{1/2} \varepsilon = \left[ \left[ \varepsilon (v^2/Q^2 + 1) \right]^{1/2}, \left[ \frac{1}{2} (1 + \varepsilon) \cos^2 \phi \right]^{1/2} \right]$$

The quantities $\varepsilon$ and $\phi$ are invariant under Lorentz boosts along the photon direction only. The calculation was performed in the laboratory (stationary proton) frame. In terms of the variables

- quark mass $= m$,
- $\hat{s} = (k + q)^2$,
- $S = (l_1 + p_r)^2$

$$\beta = \left( 1 - \frac{4m^2}{\hat{s}} \right)^{1/2}, \quad D = [4m^2 + Q^2(1 - \beta^2)]$$

Equation (4) has no $\sin \phi$ or $\sin 2\phi$ dependence; this is a consequence of parity invariance $\phi \to -\phi$. The $\cos \phi$ dependence vanishes due to the $q - \bar{q}$ symmetry, which requires invariance under $\phi \to \phi + \pi$ [6]. The $\cos 2\phi$ term is entirely mass dependent, vanishing in the $m^2 \to 0$ limit. In principle this term gives a means of experimental determination of the effective value of $m$.

3. Results and Discussion

The gluon helicity models considered are the conservative $SU(6)$, Carlitz–Kaur and Diquark, as described by Babcock et al. [3]. Of these, the first two are identical functions, differing only by normalisations. For the case of conservative $SU(6)$, the constant $c$ of equation (1) is determined by the condition

$$1 - \varepsilon = \frac{1}{2} \eta$$

corresponding to the momentum sum rule. For the Carlitz–Kaur model this normalization condition is replaced by

$$1 - \varepsilon = \frac{1}{2} \eta$$

Hence the asymmetry cannot distinguish between these two models, since the normalization constants cancel out. The difference will, however, be reflected in