A $SO(10)$ model with Majorana masses for the $v_R$'s around $10^{11}$ GeV

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Abstract. A unified gauge model is built with Higgs in $210 \oplus 126 \oplus 10$ representations and intermediate symmetry

$SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$.

The vacuum of the 210 is in a two-dimensional stratum. From the values $\sin^2 \theta_w(M_w)$ and $\frac{\alpha}{\alpha_e}(M_w)$ one determines the high scales, with the result to predict leptoquarks heavier than $10^{15}$ GeV and Majorana masses for the right-handed neutrinos around $10^{11}$ GeV.

1 Introduction

Unified theories based on $SO(10)$ [1] may have in general an intermediate symmetry larger than the standard gauge group $G \equiv SU(3) \otimes SU(2) \otimes U(1)$. This is related to the fact that at least two different irreducible representations for the Higgs are needed for the spontaneous breaking of $SO(10)$ into $G$ and the corresponding vacua have little groups larger than $G$, which is just the common symmetry of these vacua. The presence of more than one scale, corresponding to the absolute values of the VEV's of the scalars belonging to the different irreducible representations, gives more flexibility in the construction of unified theories. This flexibility is phenomenologically welcome for the well known inadequacy of the minimal unified theory based on $SU(5)$ [2] to reconcile the experimental value of $\sin^2 \theta_w$ with the lower bound on nucleon stability. A recent accurate theoretical analysis of the experiments involving neutral currents gives $\sin^2 \theta_w(M_w) = 0.228 \pm 0.004$ [3], which is slightly larger than the value predicted by the minimal $SU(5)$ model $0.214 \pm 0.004$. This gives further encouragement to look for different models.

Various possibilities, with Higgs transforming as different pairs of irreducible representations, have been considered. In all these cases the component of the VEV along one of the two irreducible representations has little group $SU(5)$: if this gets the higher value, one has the same predictions of the model based on $SU(5)$ as long as for the relationship between the values of the gauge coupling constants and the nucleon decay. Therefore it is worth to look for the other case where the other component gets the higher value. One has already discussed models with Higgs in the $45 \oplus 16$ [4], $54 \oplus 126$ [5] and $210 \oplus 126$ [6] representations with intermediate symmetries $(SU(5) \otimes U(1))$, $(SO(6) \otimes SO(4) \otimes D)$ and $(SO(6) \otimes SO(4) \otimes D)$ or $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \otimes D$ respectively.

Here we want to discuss another possibility, again with Higgs in the $210 \oplus 126$ representations, where the VEV of the 210 is in a direction $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ invariant where a positive quartic invariant vanishes.

The direction chosen lays on a two dimensional stratum and provides another counterexample to Michel's conjecture, again with a higher rank tensor (as the 210) following the general trend of those appearing in the literature [7]. A particular family of Higgs potentials involving the 210 has been recently discussed by Basecq, Meljanac and O'Raifeartaigh [8], with special attention to the problem of the stability of the breaking.

In the model presented here one may get from the experimental values of $\sin^2 \theta_w(M_w)$ and $\frac{\alpha}{\alpha_e}(M_w)$ the values of the $SO(10)$ and $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ symmetry breaking scales $M_X$ and $M_R$, which come out $\geq 1.5 \cdot 10^{15}$ GeV and
\(~5 \cdot 10^{10} \text{ GeV}\) respectively. Since the leptoquarks, which mediate nucleon decay, take their masses at the scale \(M_X\) the model is consistent with the lower limit on nucleon lifetimes. From the other side the right-handed neutrinos get their Majorana mass at the scale \(M_R\); therefore one expects, through the seesaw mechanism [9], masses for \(\nu_\tau\) and \(\nu_\mu\) of the order suggested [10] to explain the anomalies found in the experiments on atmospheric [11] and solar [12] neutrinos.

In Sect. 2 we construct a scalar potential with the desired symmetry breaking. In Sect. 3 we describe the spectrum of the scalar states and deduce the possible values of \(M_X\) and \(M_R\) from the one-loop expressions for \(\sin^2 \theta_W(M_W)\) and \(\frac{\alpha}{2\pi}(M_W)\) deduced from the renormalization group equations. In Sect. 4 we compare this model with the previous ones and give the conclusions.

2 The spontaneous symmetry breaking of \(SO(10)\)

We write, as in the previous papers [6], the potential:

\[
V(\Phi, \psi_+, \psi_-, \varphi) = V_0(||\Phi||, ||\psi||, ||\varphi||) + V_\varphi(\Phi) + V_\psi(\psi_+, \psi_-) + V_{\varphi\psi}(\Phi, \psi_+, \psi_-) + V_{\varphi\psi\varphi}(\Phi, \psi_+, \psi_-, \varphi)
\]

where \(\Phi, \psi_+, \psi_-\) and \(\varphi\) denote the 210, 126, \(\overline{126}\) and 10 representations respectively, \(V_0\) depends only on the norms and the other terms contain all the non trivial invariants. We call the moduli corresponding to the absolute minimum of \(V\):

\[
|\langle \Phi \rangle_0| = x
\]
\[
|\langle \psi \rangle_0| = \beta
\]
\[
|\langle \varphi \rangle_0| = \gamma
\]

and we assume \(\gamma < \beta < x\).

The most general form for \(V_\varphi\) has been extensively discussed in the previous works [5, 6], where one has constructed a potential which is minimum in the \(SU(5)\) invariant direction \(\psi_\pm\). We write \(V_\varphi\) and \(V_{\varphi\psi}\):

\[
V_\varphi(\Phi) = A ||(\Phi \delta)_{45}|| + B ||(\Phi \delta)_{54}|| + C ||(\Phi \delta)_{210}|| - D z_1(\Phi \delta)_{210} \cdot \Phi
\]

\[
V_{\varphi\psi}(\Phi) = f_1 \cos \theta_3(\psi_+ \Phi)_{120} + \sin \theta_3 e^{i\eta_3}(\psi_- \Phi)_{120}
\]
\[
+ f_2 \cos \theta_4(\psi_+ \Phi)_{120} + \sin \theta_4 e^{i\eta_4}(\psi_- \Phi)_{120}
\]
\[
+ f_3 \cos \theta_5(\psi_+ \Phi)_{320} + \sin \theta_5 e^{i\eta_5}(\psi_- \Phi)_{320}
\]
\[
+ f_4+ ||(\psi_+ \Phi)_{126} + \frac{x}{10\sqrt{5}} (3+\sqrt{2}) \psi_+ ||
\]
\[
+ f_4- ||(\psi_+ \Phi)_{126} + \frac{x}{10\sqrt{5}} (3-\sqrt{2}) \psi_+ ||
\]
\[
+ f_5(\psi_+ \varphi)_{45} - (\Phi \delta)_{45}
\]

We want to find the absolute minimum of \(V_\varphi + V_\varphi\psi\) in the direction:

\[
\Phi = \frac{\Phi_{1234} + \Phi_{1256} + \Phi_{3456} + \sqrt{2}\Phi_{789}_{10}}{\sqrt{5}}
\]

with little group \(SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_B-L\). From the \(SO(10)\) Clebsch-Gordan rule

\[
\sum_{I} z_I \left( \begin{array}{ccc} 210 & 210 & 54 \\ a & b & c & d \\ e & f & g & h \\ 11 & 11 \\ \end{array} \right)
\]

where \(\sum z_I = 0\) since the 54 is the traceless symmetric rank 2 tensor and the dots mean antisymmetrization in \(abcd\) and \(efgh\), it follows that \(||\Phi \delta)_{54}|| = 0\) so the positive definite invariant \(||\Phi \delta)_{54}||\) vanishes in the direction \(\Phi\). Therefore a large and positive value for \(B\) would favour the direction \(\Phi\) to be on the absolute minimum of \(V_\varphi\).

However the invariant \(||\Phi \delta)_{54}||\) vanishes also in other directions; it would depend on the parameters \(A, C, D\) in (3) and \(f_i\) in (4) whether the absolute minimum of \(V_\varphi + V_\varphi\psi\) would fall in the direction \(\Phi\). The first three invariants in (4), which are positive definite, vanish if \(\Phi\) and \(\psi_\pm\) have a common \(SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_B\) invariance algebra, since the 10, 120 and 320 representations do not contain any singlet under that algebra. Therefore if \(f_1, f_2\) and \(f_3\) are positive and large enough, we may restrict the search for the absolute minimum of \(V_\varphi\) to \(SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_B\) invariant directions, since it is reasonable to expect that at least one of the first three terms in (4) would receive a positive contribution in each different direction.

There are three independent \(SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_B\) singlets in the 210 representation and we shall look for the absolute minimum of \(V_\varphi(\Phi)\) on the vector space spanned by them: