QCD Sum Rules and Pion Wave Functions

M.J. Lavelle
Blackett Laboratory, Imperial College, LONDON SW7 2BZ, UK

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Abstract. We consider light cone sum rules for a vertex function involving a pion state. These incorporate radiative corrections, continuum effects and power corrections; the latter depend on the non-asymptotic form of a higher twist component of the pion wave function. We derive restrictions on this component from sum rules for two-point functions and propose a model wave function for it. Finally we analyse our vertex sum rules in the light of this information and find results for the lowest twist component in good agreement with those already obtained from two-point function sum rules with derivative currents.

1. Introduction

The recent progress in our understanding of exclusive processes in perturbative QCD [1, 2] is hampered by our lack of knowledge of the non-asymptotic form of the hadronic wave functions involved. It has been suggested that these (universal) non-perturbative objects could be directly determined from experiment [3, 4] or from QCD sum rules for two-point functions. In the latter case a great deal of work has been done by Chernyak and Zhitnitsky [2, 5, 6], who find wave functions differing greatly from their asymptotic form (Fig. 1). The concave nature of their results goes against one's initial prejudice, as voiced, for example, by Ioffe [7], who claims that the true non-asymptotic form of the wave function, though perhaps broader, should be convex.

In the face of this debate, and whilst we still lack any experimental checks, it seems desirable to independently check the results of Chernyak and Zhitnitsky for the pion wave function. A natural technique to use for this is the vertex function sum rule method [8], which has already been used to check the results of [6] for the proton wave function [9]. Indeed Craigie and Stern in [8] by considering the vertex function we shall use below, obtained results agreeing with Chernyak and Zhitnitsky. However, their analysis involved the use of two sum rules (one with the ABJ anomaly [10] to provide a subtraction constant and one without any subtraction), and combining these leads to inconsistencies.

For these reasons we now proceed to consider the following vertex function which probes the pion wave function [8]:

\[ i \langle 0 | T \left( J_{\mu}^\pi(x) J_{\nu}^\pi(0) \right) | 0 \rangle \],

where \( J_{\mu}^\pi \) is the electromagnetic current. In Sect. 2 after the preliminary definitions we consider the lowest order QCD result [8], the radiative corrections and the power corrections. We also consider an ansatz for the phenomenological side of our sum rules. Section 3 removes the major uncertainty in our sum rules, i.e. the lack of knowledge of the non-asymptotic form of one of the higher twist wave functions contributing to the power corrections. We consider a set of two-point functions and find the values of the first few moments of this wave function. We then use this information in Sect. 4 to analyse our sum rules for the vertex function and thus estimate the first non-trivial moment of the lowest twist pion wave function, which is sufficient to differentiate between a concave and a convex wave function. Finally in Sect. 5 we briefly discuss the results obtained.
2. The Vertex Function

Our vertex function is defined as:

\[ i \int d^4 x e^{i p x} \langle \pi(p) | T(J_\mu(z) J_\nu(0)) | 0 \rangle = \epsilon_{\mu\nu\rho\sigma} q^\rho p^\sigma T(q_1^2, q_2^2) \]  

(2.1)

where \( |\pi(p)\rangle \) is the pion state with momentum \( p \)
and

\[ q_1^2 = (q + \frac{1}{2} p)^2, \quad q_2^2 = (q - \frac{1}{2} p)^2 \]

and the electromagnetic current \( J_\mu \) is:

\[ J_\mu = \frac{1}{2} \bar{u} \gamma_\mu u - \frac{1}{2} i d \gamma_\mu d. \]

The lowest twist pion wave function, \( \phi_\pi \), is defined as [5]

\[ \langle 0 | \bar{u}(z) \gamma_5 \gamma_\mu | \pi(q) \rangle = i f_\pi q \phi_\pi(q, \mu^2) + \cdots \]

(2.2)

where \( \mu^2 \) is the renormalisation point. The moments of the wave function are defined as

\[ \langle \xi^{2n} \rangle = \int_0^1 d\xi \xi^{2n} \phi_\pi(\xi^2, \mu^2), \quad \langle \xi^0 \rangle = 1. \]

Hence from lowest order QCD (Fig. 2) we get (in the chiral limit) the result [8]:

\[ T(q_1^2, q_2^2) \sim \int d\xi \phi(\xi, Q^2)/(1 - \omega \xi) \]

\[ = f_\pi [1 + \omega^2 \xi^2 + \cdots] \]

(2.3)

where

\[ \omega = \frac{q_1^2 - q_2^2}{q_1^2 + q_2^2}, \quad Q^2 = -q^2. \]

The first set of corrections considered are the radiative corrections (Fig. 3). These have previously been considered for the specific values of \( \omega = 0 \) and 1 by Del Aguila and Chase [11] and Voloshin [12] who agreed for \( \omega = 1 \) but disagreed for \( \omega = 0 \) by a factor of 5/6. Braaten [13] has calculated these diagrams for general \( \omega \) and agrees with [11] in the \( \omega \to 0 \) limit. We have repeated this calculation and agree with the results of [13] (and therefore [11]). The final answer (see Appendix) is, of course, UV finite and we subtract the IR divergences to obtain the result we employ (necessary only for \( \omega \) non-zero). Modulo the lowest order factor the correction is therefore given by:

\[ \omega^0 : -\frac{2}{\pi} \]

\[ \omega^2 : -\frac{5}{12\pi} (\xi^2 - 1). \]

The next type of corrections to be considered are the power corrections (Fig. 4) due to the higher twist components of the pion [2]. These have been studied by Gorsky [14], who considered the same vertex. Of the three leading corrections only two contribute, as the twist 3, 2-particle pion wave function coming from \( \bar{u} \gamma_5 d \) gives zero from the trace (Fig. 2, 3). The two leading higher twist corrections are dependent on the wave functions \( \phi_2 \) and \( \phi_3 \), which are defined by [2]:

\[ \langle \pi^- (p) | u(y) \bar{u}(0) G_{\pi\pi}(z) | \pi^0 (0) \rangle = i \sqrt{2} p_\mu C f_\pi \phi_2(y p, \mu^2) \]

(2.5)

and

\[ \langle \pi^0 (p) | u(y) \bar{u}(0) G_{\pi\pi}(z) | \pi^0 (0) \rangle = -i f_\pi C' (p_\mu d_{\pi\pi} - p_\nu d_{\pi\pi}) \]

\[ \cdot \int [dx] \phi_3(x, \mu^2)e^{i\lambda(x_1 y + x_2 z)} \]

(2.6)

where \( [dx] = dx_1 dx_2 dx_3 \delta(1 - \Sigma x_i) \).

We now have all the leading corrections to the QCD side of our vertex function.

For the phenomenological side of the sum rule we take a light cone dispersion relation along a line in