On the BFT-BFV quantization of gauge invariant systems with linear second class constraints

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Abstract. We study the Hamiltonian path integral formulation for generic systems with first class and linear second class constraints.

1 Introduction

The quantization of Hamiltonian systems with both first and second class constraints [1] can be achieved in several ways. Second class constraints are conveniently treated at classical level with the use of Dirac brackets (DB's) [1] [2], which can be in principle replaced by (anti) commutators in the process of canonical quantization. The physical states which describe the relevant solutions of the theory are selected among all the other possible states, by imposing that they are to be annihilated by the first class constraints, written as operators at the quantum level. The second class constraints are supposed to value as operatorial identities. Even if there are no ordering problems, which could introduce central terms in the first class algebra, in general it is not an easy task to represent the fundamental variables of the theory as operators, due to the usual complexity of DB structures [2].

This difficult is essentially kept when other methods of quantization are taken in account, as the BRST operatorial quantization procedure [3] or its Hamiltonian functional counterpart, the Batalin, Fradkin and Vilkovisky (BFV) method [4], later adapted to consider also second class constraints by Fradkin and Fradkina (FF) [5].

In an attempt to unify the methods of quantization for systems with both classes of constraints (and at the same time define extended algebras in terms of Poisson brackets (PB's) in place of the original ones constructed with the aid of DB's), Batalin, Fradkin and Tyutin (BFT) [6] have develop a systematic algorithm which implements the Abelian conversion of the second class constraints of the theory under consideration.

In a recent work by the author and Das [7], it has been shown that when the second class constraints are linear in the canonical variables, the BFT algorithm is implemented by a proper shift in those variables, permitting the Abelian conversion to be done in a generic way. We observe that a great amount of interesting works has recently been done concerning the application of the BFT formalism to Hamiltonian systems where linear second class constraints are present [8]. The obtainment of general results which have to be satisfied by all of such systems becomes then a relevant point. Actually it can be shown [9] that the models treated in ref. [8] are BFT-extended just by the shift procedure described in [7] and improved in the present work. At the same time those systems [8] can be quantized along the functional methods to be introduced in section 4. Incidentally, we observe that the collective field formalism, due to Alfaro and Damgaard [10], very powerful in deriving the BV [11] formalism from the BRST one, as well as in the obtainment of general Ward identities, can be implemented in the scope of Abelian converted linear second class constraints originally associated to first order Lagrangians [7].

Our paper is organized in the following manner: In section 2 we give a review of Hamiltonian systems with both first and second class constraints and establish the notation to be used throughout the paper. Following this we present some of the results due to Fradkin and Fradkina [5] concerning the functional quantization of such systems. In section 3 we extend the results of [7], by showing the general correspondence between the constraint algebras constructed with the original phase space variables and those obtained with the aid of the BFT variables. By using the results of section 3, the functional quantization of gauge invariant systems with Abelian converted linear second class constraints is studied in section 4. We also show that in the unitary gauge the results of FF are reobtained. Section 5 is devoted to some final comments and conclusions.

2 Hamiltonian systems with first and second class constraints

Consider a Hamiltonian system evolving in a phase space $P$ described by the canonical set of variables
where \( f^{\mu}_{\nu} \) is in general a singular matrix. It can be shown [2] that on the second class constraint surface, \( f^{\mu}_{\nu} \) induces a two-form which actually is regular. This corresponds, however, to a reduced phase space treatment which can bring explicitly breaking of covariance as well as the appearance of non-linearities.

By using expression (5), (4) can be written as
\[
\{A, B\}_* = \frac{\partial A}{\partial y^\mu} f^{\mu}_{\nu} \frac{\partial B}{\partial y^\nu}.
\]

The introduction of DB's permits to write relations (3) in an alternative way as
\[
\{\chi_\alpha, \chi_\beta\}_* = 0,
\]
\[
\{\chi_\alpha, \gamma_\alpha\}_* = 0,
\]
\[
\{\gamma_\alpha, \gamma_\beta\}_* = C_{ab} \gamma_c + T_{ab} \chi_c \chi_\beta,
\]
\[
\{\gamma_\alpha, H_\beta\}_* = V_{ab} \chi_\beta + V_{ab} \chi_a \chi_\beta,
\]
\[
\{\chi_\alpha, H_\beta\}_* = 0,
\]
where the structure functions appearing above in general are not the same as those appearing in (3).

The BFV [4] quantization of a system like the one appearing in (7) can be done along the lines described by FF [5]. The vacuum functional is defined by
\[
Z_\Psi = \int [d\gamma^\mu] [d\gamma_\alpha] [dP_a] [d\eta^a] [d\chi_\alpha] \frac{1}{\det(A)} \delta(\chi_\alpha) \delta(\gamma_\alpha) \delta(\gamma_\beta) \delta(\chi_\beta) \delta(P_a) \delta(\eta^a) \exp\{i \int dt \left[ \frac{1}{2} \gamma^\mu f^{\mu}_{\nu} \gamma^\nu + P_a \eta^a - H_{\Psi} \right] \}.
\]

The ghosts have odd Grassmanian parity (in our formulation ghosts are fermionic quantities) and satisfy the usual PB relations
\[
\{\gamma^a, \eta^b\}_* = -\delta_a^b.
\]
Also
\[
H_\Psi = H_1 + \{\Psi, \Omega_1\}_*.
\]
is the BRST invariant gauge-fixed Hamiltonian, which depends explicitly on the gauge fermion \( \Psi \). The proper form of \( \Omega \) and \( H_1 \) depends on the rank of the specific theory considered and can be determined [4] [5] when one imposes the nilpotency of the BRST generator,
\[
\{\Omega, \Omega_1\}_* = 0,
\]
as well as the involution relation
\[
\{H_1, \Omega_1\}_* = 0,
\]