(\eta - \eta' - \iota)-mixing, decay and production mechanisms

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Abstract. We present a phenomenological model for meson-glueball mixing and apply it to the mixing of the pseudoscalar mesons \(\eta\) and \(\eta'\) with the \(\iota(1440)\)-particle as the prime candidate for a \(0^-\)-glueball. Using the MIT bag model framework, our model incorporates, among other elements, configuration mixing effects due to virtually excited states, as well as mixing of quark and gluon components. We evaluate the portions of quark-antiquark and two-gluon admixtures in \(\eta, \eta'\) and \(\iota\). The results are used to study various decay and production processes involving these particles. In particular, we calculate the radiative and the \(2\gamma\)-decay widths of the light mesons and make predictions for the widths of the \(\iota\) in these channels. Furthermore, production mechanisms of the light mesons in electromagnetic and hadronic \(J/\psi\)-decays are used to evaluate the branching ratios for the production of the \(\iota\) in various \(J/\psi\)-transitions. Our results are generally in reasonable agreement with experimental data. Especially we predict a rather large width for the decay \(\iota \rightarrow \rho \gamma\) and a sizable production rate for the \(\iota\) in the transition \(J/\psi \rightarrow \omega \eta\).

1 Introduction

Besides the normal mesons, i.e. bound states of constituent quarks \((q\bar{q})\), the gauge theory of strong interactions, quantum chromodynamics (QCD), predicts the existence of bound states consisting of two or more constituent gluons (glueballs). A unique experimental identification of such new states should be a further strong argument for the validity of QCD as the theory of strong interactions. However, the search for glueballs is hampered by the lack of a readily recognizable signature for gluonic states. In general, the quantum numbers of glueballs are the same as for \(q\bar{q}\)-mesons; furthermore the masses of the lowest states are predicted to lie within the 1–2 GeV region, which is densely populated by \(q\bar{q}\)-states [1–3]. Thus it is very likely that these glueballs will mix with neutral \(q\bar{q}\)-mesons, a property that makes it even more difficult to distinguish them from conventional quarkonic states. In particular, the 'prima facie' glueball candidate, the \(\iota(1440)\) with \(J^{PC} = 0^-\), is expected to mix with the pseudoscalar mesons \(\eta(549)\) and \(\eta'(958)\) and there is experimental evidence that this is indeed the case [2, 4].

We have carried out an extensive analysis of glueballs and their decay and production properties within a specific model, namely an extended version of the MIT bag model [5], incorporating meson-glueball mixing. It is our aim to find significant and characteristic features of glueballs that allow for a discrimination from conventional quark mesons. In this paper we will in particular consider the pseudoscalar mixing between \(\eta, \eta'\) and the \(\iota\)-particle. We will give definite predictions for the radiative decays and the \(2\gamma\)-decays of the \(\iota\), as well as for branching ratios for its production in various \(J/\psi\)-decays.

The paper is organized as follows: In Sect. 2 we briefly introduce our model and in Sect. 3 we explain how meson-glueball mixing is described within this model. The applications of this framework to the mesonic decays is subject of Sect. 4, while Sect. 5 deals with the production mechanisms of \(\eta, \eta'\) and \(\iota\) in the radiative and hadronic \(J/\psi\)-decays. Finally, we present a discussion of the results and concluding remarks. The Appendix contains a brief discussion of a study that we carried out for the mass spectra and static properties of the low lying hadrons and glueballs to test the model and fix its parameters. We present some of the results on which the considerations of this paper are based.

The properties of glueballs and their relationship with conventional mesons have of course also been discussed by others, within very different approaches. In the literature, there are various theoretical works that make predictions for the masses of low-lying glueballs, based on e.g. lattice calculations, potential models and also the bag model framework. Reference [1] presents a general overview of the experimental and theoretical state of...
knowledge concerning the research on glueballs and also provides a comprehensive list of references.

A comparison of results from different theoretical models can be found in [3]. For related bag model calculations, we refer to [6–9]. The mixing of \( \eta, \eta' \) and \( t \) has been investigated by numerous authors, however, most of these studies are based on analyses of experimental data, e.g. [4, 15, 16, 25] and references therein. Meson-glueball mixing within the MIT bag model framework has been considered in [10] and [11]. Finally, the papers of [14] present a detailed discussion of decay and production properties of \( \eta, \eta' \) and \( t \) based on a non-relativistic quark model with a harmonic oscillator potential. It is interesting to note, that the results given there, are generally in good agreement with ours, although we do employ a rather different approach.

2 The model

Inspired by the approach of Carlson et al. [6], our model that allows for a consistent and unified study of the low-lying mesons, baryons and glueballs, including possible mixing properties, is based on the MIT bag model framework. In our analysis we use the same parameters for glueballs as for the mesons and baryons.

In comparison with the original MIT model [5], our approach contains several additional elements, that in part have been incorporated in bag model calculations also by others, namely

- configuration mixing of various bag states,
- self-energy contributions of the constituents [6–8],
- running coupling constant [6, 7],
- corrections for the center of mass motion of the bag [6–9].

Since in the coming sections we will be concerned with properties and consequences of meson-glueball mixing, we outline our basic concept only for the case of \( q\bar{q} \)-mesons and \( gg \)-glueballs that are composed of two constituent gluons (see also Appendix). The baryon sector and other more complex quark and gluon systems can be treated similarly in this framework.

The strategy is the following: We start from unperturbed bag eigenstates which are color-singlets [1] with definite \( J^P_c \), i.e.

\[
|q_1\bar{q}_2\rangle_s \equiv |a_1^{\dagger}b_2^{\dagger}|^{(J^P_c;1)}|0\rangle \quad (1a)
\]

\[
|g_1g_2\rangle_s \equiv |c_1^{\dagger}c_2^{\dagger}|^{(J^P_c;1)}|0\rangle \quad (1b)
\]

where \( a^{\dagger}, b^{\dagger} \) denote the creation operators for a quark, respectively an antiquark and the \( c^{\dagger} \) are creation operators for gluons with either transverse electric (TE) or transverse magnetic (TM) polarisations, acting on the perturbative vacuum \( |0\rangle \) inside the bag. The numbers 1 and 2 stand for complete sets of quantum numbers of the one-particle states, and the index \( z \) for the two-particle states denotes a certain 'configuration', corresponding to the various possibilities of coupling the constituents 1 and 2 to a composite system.

With these composite states, we then calculate the energy shift to first order in the coupling constant \( \varepsilon_s \), using the formalism of covariant perturbation theory for QCD in a static cavity of [12], i.e. we evaluate the interaction amplitudes corresponding to the tree diagrams shown in Fig. 1a. Also, we take into account self-energy contributions to \( O(\varepsilon_s) \) for both, quarks and gluons, which are represented by the loop diagrams in Fig. 1b. However, since up to now there exists no unambiguous method for a consistent calculation of general loop diagrams in the bag, we rather parametrize the self-energies.

Thus, we express the energy of the constituents in the bag including \( O(\varepsilon_s) \) interactions as

\[
E_{QCD}^{(q\bar{q})} = \frac{\varepsilon_1 + \varepsilon_2 + \varepsilon_s(\hat{A} \hat{R})}{R} + \varepsilon_s(\hat{q}\tilde{q}) \quad (2a)
\]

\[
E_{QCD}^{(gg)} = \frac{\omega_1 + \omega_2 + \varepsilon_s(\hat{A} \hat{R})}{R} + \varepsilon_s(\hat{g}\tilde{g}) \quad (2b)
\]

Fig. 1 a–c. Feynman diagrams corresponding to interactions in first order of the coupling constant \( \varepsilon_s \) (solid lines represent quarks, wiggly lines are gluons). a Tree diagrams: quark-antiquark interaction via gluon exchange \( EX(q\bar{q}) \), and pair annihilation \( AN(q\bar{q}) \), and below, gluon–gluon interaction via gluon exchange \( EX(gg) \), pair annihilation \( AN(gg) \), and elementary point vertex \( PV(gg) \), respectively. b Loop diagrams: quark self-energy \( S(q) \), and antiquark self-energy \( S(\bar{q}) \), contributing to \( q\bar{q} \)-system and gluon self-energy diagrams \( S(g) \), for the \( gg \)-system. c \( (q\bar{q} \rightarrow gg) \) interactions that contribute to lowest order meson-glueball mixing, \( l(q\bar{q} \rightarrow gg) \) and \( l(gg \rightarrow q\bar{q}) \).