Dynamical supersymmetry breaking and gauge anomalies

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Abstract. Some aspects of supersymmetric gauge theories and discussed. It is shown that dynamical supersymmetry breaking does not occur in supersymmetric QED in higher dimensions. The cancellation of both local (perturbative) and global (non-perturbative) gauge anomalies are also discussed in supersymmetric gauge theories. We argue that there is no dynamical supersymmetry breaking in higher dimensions in any supersymmetric gauge theories free of gauge anomalies. It is also shown that for supersymmetric gauge theories in higher dimensions with a compact connected simple gauge group, when the local anomaly-free condition is satisfied, there can be at most a possible $Z_2$ global gauge anomaly in extended supersymmetric $SO(10)$ or $Spin(10)$ gauge theories in $D = 10$ dimensions containing additional Weyl fermions in a representation of $SO(10)$ or $Spin(10)$. In four dimensions with local anomaly-free condition satisfied, the only possible global gauge anomalies in supersymmetric gauge theories are $Z_2$ global gauge anomalies for extended supersymmetric $SP(2N)$ ($N = \text{rank}$) gauge theories containing additional Weyl fermions in a representation of $SP(2N)$ with an odd 2nd-order Dynkin index.

1 Introduction

Since the discoveries of Yang–Mills theory [1] and supersymmetry [2], supersymmetric gauge theories have been one of the most interesting ideas in elementary particle physics [3]. Gauge theories have played one of the most important roles in the unification of fundamental interactions [4] in nature. It is also known that the open superstring theory [5] can be approximated at low energy by a supersymmetric gauge theory. Supersymmetric gauge theories are of fundamental interest in particle physics beyond the standard model.

In the construction of realistic models to describe nature, supersymmetry must be spontaneously broken if it plays a role in the models, due to the fact that degenerate Bose–Fermi multiplets are not observed. Therefore, the study of the possibilities of spontaneous supersymmetry breaking is crucial in supersymmetric gauge theories.

Witten has given extensive discussions about the dynamical breaking of supersymmetry by non-perturbative effects [6, 7], and this is one of the main topics we are interested in here.

In global supersymmetric theories, a common feature is that the Hamiltonian $H$ is the sum of the squares of the supersymmetry charges which are hermitian. This implies that supersymmetry is spontaneously broken if and only if the ground state has energy greater than zero. According to this, it is a useful strategy to study the low-lying states in the spectrum of the Hamiltonian in order to determine the possibilities of spontaneous supersymmetry breaking. Because whether the zero-energy ground state exists may be determined explicitly by this strategy. This strategy turns out to be very useful to the study of dynamical supersymmetry breaking, in which supersymmetry is preserved in the classical theory but broken by the non-zero energy of ground state created in the quantum dynamics. For the analysis of zero energy states, Witten introduced an operator $(-1)^F$ whose trace can be written as [7]

$$\text{Tr}((-1)^F) = n_E^{E=0} - n_E^{E=0},$$

(1.1)

where $n_E^{E=0}$ and $n_E^{E=0}$ are the numbers of zero-energy bosonic and fermionic states respectively. If $\text{Tr}((-1)^F) \neq 0$, then there exists at least one zero-energy ground state, the supersymmetry is not spontaneously broken. The usefulness of (1.1) lies in the fact that it can be regarded as the index [8] of an operator $M$, when a supersymmetry change $Q$ is written in the form of

$$Q = \begin{pmatrix} 0 & M^+ \\ M & 0 \end{pmatrix},$$

(1.2)

by splitting the Hilbert space of the theory into bosonic and fermionic subspaces $H_B$ and $H_F$ with the states arranged in the form $|B \rangle$. Since the index of an operator is invariant under continuous deformations, the $\text{Tr}((-1)^F)$ is independent of the parameters of the theory if the
different set of parameters can be reached to each other by continuous deformations, for example, by conjugate transformations. In certain interesting classes of theories including supersymmetric gauge theories in four dimensions, as shown by Witten the index (1.1) is indeed independent the parameters of theories. Therefore, for those theories, Tr(−1)f can be calculated in a convenient limit, such as small volume, large bare mass, and weak coupling.

More generally, one can calculate the trace

\[ \text{Tr}(−1)f(x) = \sum_{i} f(\lambda) \text{Tr}(−1)f_{\lambda}, \]  

(1.3)

where \( X \) is any operator that commutes with the supersymmetry charges \( [X, Q_{\pm}] = 0 \), and thus commutes with the hamiltonian also. The \( P_{\lambda} \) denotes the projection from the Hilbert space onto its subspace with \( x = \lambda \) for a definite eigenvalue \( \lambda \). It turns out that the generalized formula (1.3) is particularly useful in supersymmetric gauge theories, where one calculates the trace \( \text{Tr}(−1)f \) in a physical eigen-subspace for some operator \( X \). If \( \text{Tr}(−1)f(x) \) is non-zero for some choice of \( f(x) \), then supersymmetry is not spontaneously broken, namely the dynamical supersymmetry breaking does not occur. In order to determine whether the index is non-zero, one may expect to restrict to the minimum supersymmetric gauge theories, although additional fields may be present more generally [7].

One of the main features of this paper is that we apply Witten’s method of calculating the trace (1.3) to the supersymmetric gauge theories in higher dimensions. To proceed, we will now briefly describe the minimum supersymmetric gauge theories in higher dimensions.

A minimal supersymmetric gauge theory consists of the gauge field \( A_{\mu} \) and a spinor \( \psi \). The index \( a \) corresponds to the generators of the gauge group \( G \) with Lie algebra \( L(G) \). We assume \( G \) is compact and connected. Since \( A_{\mu} \) and \( \psi \) transform into each other under supersymmetry transformations, it is necessary that the spinor fields \( \psi^a \) (\( a = 1, 2, ..., \dim L(G) \)) also form the adjoint representation of \( G \) under the gauge transformations.

The Lagrangian density is given by

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \bar{\psi} \Gamma^\mu D_\mu \psi, \]  

(1.4)

with the action in \( D \) dimensions

\[ S = \int d^Dx \mathcal{L}, \]  

(1.5)

where

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g C_{abc} A^b_\mu A^c_\nu, \]  

(1.6)

\[ D_\mu \psi = \partial_\mu \psi + g C_{abc} A^b_\mu \psi, \]  

(1.7)

with gauge coupling constant \( g \). The \( C_{abc} \) are the structure constants of \( L(G) \), determined by \( [L_a, L_b] = i C_{abc} L_c \) with \( \{L_a | a = 1, 2, ..., \dim L(G) \} \) as a basis of the Lie algebra.

The number of physical fermionic modes corresponding to the spinor field is a power of two depending on the spacetime dimension and the type of spinor (for example, Dirac, Majorana, Weyl, both Majorana and Weyl). The gauge potential \( A_\mu \) corresponds to \( D - 2 \) physical modes for a given generator in \( L(G) \). Since supersymmetry requires that the number of bosonic and fermionic physical modes should be equal, only in some specific dimensions there can be supersymmetric gauge theories. The \( D - 2 \) is a power of two for \( D = 3, 4, 6, 10 \) etc. For \( D > 10 \), the number of fermionic physical modes greatly exceeds that of the vector fields. Although additional matter multiplets may be included, it is believed that there exist no supersymmetric gauge theories in dimensions \( D > 10 \).

It is shown [9] that the theory described by (1.4)–(1.7) is indeed supersymmetric in \( D = 3, 4, 6, 10 \). The supersymmetry transformations that leave (1.5) invariant are

\[ \delta A^a_\mu = \frac{1}{2} [\bar{\epsilon} \Gamma^\mu \psi^a - \bar{\psi}^a \Gamma^\mu \epsilon], \]  

(1.8)

\[ \delta \psi^a = \frac{1}{2} \sum_{\mu} F^{\mu a}, \]  

(1.9)

\[ \delta \bar{\psi}^a = -\frac{1}{2} \bar{\epsilon} \sum_{\mu} F^{\mu a}, \]  

(1.10)

with

\[ \sum_{\mu} = \frac{1}{2} [\Gamma^\mu, \Gamma^\nu]. \]  

(1.11)

Furthermore, the spinors are Majorana in \( D = 3 \), Majorana or Weyl in \( D = 4 \), Weyl in \( D = 6 \), Majorana and Weyl in \( D = 10 \) respectively. In the case of Majorana spinors in \( D = 3, 4 \) or \( 10 \), (1.8) can be rewritten in the simpler form

\[ \delta A^a_\mu = i \bar{\epsilon} \Gamma^\mu \psi^a. \]  

(1.12)

In the case of \( D = 6 \) it is necessary to keep both terms.

The conserved supersymmetry current in the theory is

\[ S_\mu = \sum_{\mu} F^{\mu a} \Gamma^\mu \psi^a. \]  

(1.13)

Since any compact and connected Lie group is locally isomorphic to \( (U(1))^K \otimes H_1 \otimes \cdots \otimes H_L \) (\( K \geq 0, L > 0 \) and \( H_i (i = 1, \ldots, L) \) are compact connected simple Lie groups, our study here will be essentially for two different cases, namely a supersymmetric \( U(1) \) gauge theory and a supersymmetric non-abelian gauge theory with a compact connected simple gauge group. It is also well known that the consistency of a gauge theory requires the absence of gauge anomalies. Therefore, we need to consider the possible gauge anomalies also in the supersymmetric gauge theories, and this will be the another feature of the paper.

Our paper will be organized as following. We will first show in the next section that there is no dynamical supersymmetry breaking in an abelian gauge theory in six and ten dimensions. In the Sect. 3, we will argue for the same conclusion for the non-abelian gauge theories. The Sect. 4 will be for the discussions about the possible gauge anomalies in higher-dimensional supersymmetric gauge theories, and the relationship to the spontaneous supersymmetry breaking. We will summarize the conclusions of this paper in the Sect. 5.