A comment on form factor mass singularities in flavor-changing neutral currents

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Received 7 May 1991

Abstract. Flavor-changing effective vertices \( q_lq_h V^0 \), where \( V^0 \) represents a neutral gauge boson \( (\gamma, Z^0, g) \), involving a heavy external quark, are discussed within the standard model at one-loop level and second-order approximation in external momenta and masses: the logarithmic singular terms in the form factors at vanishing mass of the internal quark in the loop have to be replaced by pieces coming from next order in external momenta. Implications in the \( b \rightarrow d + X \) penguin transitions are commented.

The possibility of testing the Standard Model (SM) of the electroweak interactions and its possible extensions, as well as non-conventional physics, has increased in recent years the interest in rare decays of particles. The fact that such processes occur via diagrams involving particles like the gauge boson \( W \) and its possible extensions, or the yet undiscovered top quark, makes them especially suitable as complementary tests of other searches like those performed at collider physics. In particular, neutral gauge bosons \( (\gamma, Z^0 \text{ and } g) \) couple to quarks with different flavors at lowest order through one-loop level diagrams, giving rise to flavor-changing neutral current (FCNC) transitions.

Among rare processes, FCNC decays of \( B \) mesons have the intrinsic advantage over others (like kaon decays) of the relative large mass of the constituent \( b \) quark, offering the reasonable option of neglecting QCD effects at a first approximation and rendering credible a spectator model. Furthermore, in the usual low energy limit, one neglects external fermion masses and momenta whenever and as early as possible to simplify the calculation*. However, although such approximation is in general well-founded, a logarithmic singularity arises in the flavor-changing (FC) vertex amplitude when the mass of the internal fermion of the loop is set to zero. In fact as we shall see this a manifestation of a careless low energy limiting process performed at second-order in external momenta.

In our discussion we will essentially follow the framework and notation employed in the Barroso and Soares' paper [2], where they present an exact calculation within the SM of the FC vertices \( q_lq_h V^0 \) with \( V^0 = \gamma, Z^0 \) in the 't Hooft–Feynman gauge at one-loop level using the on-shell renormalization scheme*. Throughout this paper we will not discuss QCD corrections to the effective vertex, and we will impose the following constraints: i) on-shell external quarks, ii) the mass of the light external quark \( q_l \) is set to zero. Then, both FC renormalized vertices \( q_lq_h \gamma, q_lq_h Z^0 \) can be expressed as

\[
\hat{\mathcal{E}}^\mu_{\text{ren}}|_{\text{on-shell}} = C_s [\alpha_1 q^\mu + \alpha_2 q^\mu + \alpha_3 i q^\mu q_3] L,
\]

where

\[
\alpha_1 = \frac{m_q^2}{2} A_3 + m_b A_4 + A_{11} - \frac{1}{2} A_{12} + A_{13},
\]

\[
\alpha_2 = \frac{m_q^2}{2} A_3 + A_6 + q^2 A_7 + \frac{m_q^2 - q^2}{2} A_8 + \frac{m_b}{2} A_{12},
\]

\[
\alpha_3 = \frac{m_b}{2} A_3 - A_{11} - \frac{1}{2} A_{12},
\]

and

\[
C_s = \left[ \frac{e}{-g_\gamma} \right] \left( \frac{g^2}{4\pi^2} \right) V_F,
\]

the upper (lower) line corresponding to the \( \gamma(Z^0) \) case, and \( L \) stands for the left projection operator. We denote by \( p \) and \( q \) the incoming momenta of the light \( q_l \) quark and the neutral gauge boson \( V^0 \) respectively (Fig. 1a). For external down-type quarks in the vertex, the Kobayashi–Maskawa coefficient is \( V = V^\nu \). The \( A_s \) coefficients depend on the momenta of the external particles, and on the masses and quantum numbers of the external and internal quarks. Their full expressions

* Certain computer evaluations of the one-loop FCNC are available at present [1], but sometimes may hide a more intuitive approach

* In what matters in this paper the extension to the \( q_lq_h \gamma \) vertex is straightforward [3]
Fig. 1a–c. $q_iq_jV^0$ vertex ($q = p' - p$): a The blob represents 10 one-loop diagrams in the 't Hooft–Feynman gauge for $V^0 = \gamma, Z^0$, and 6 one-loop diagrams for $V^0 = g$; b and c two of these diagrams involving the gauge boson $W$ and the charged unphysical Higgs $\sigma$ respectively; "i" labels the internal flavor.

will not be reproduced here but are available in [2] and for the photon vertex can also be found up to minor notation differences in [4] (in both, they are presented as Feynman parametric integrals). Instead, we will pick up the relevant terms giving rise to the logarithmic singularity discussed in this paper. Let us remark that the right-handed piece of the analog expression of (1) in [2] has disappeared since we have set $m_i = 0$.

In particular, for each intermediate flavor "i" the $q_iq_j\gamma$ vertex can be written as

$$F_i(q^2 = 0) = \frac{1}{(4\pi)^2} e^2 \frac{g^2}{2M_w^2} V_i \cdot \bar{q}_i \gamma \left[ F_1(q^2) \gamma^\mu - q^\mu \gamma \right] L - m_i F_2(q^2) \gamma^\mu q_i L q_i. \quad (3)$$

The piece containing the $F_1$ form factor represents an electric (charge-radius) transition and does not contribute for a real photon ($q^2 = 0$). The piece containing the $F_2$ form factor corresponds to a magnetic transition and contributes in the latter process. In the limit of vanishing external masses and momenta, one recovers the well-known result of Inami Lim [5] at lowest (second) order in external momenta and masses for the effective $q_iq_j\gamma$ vertex. Introducing $x_i = m_i^2/M_w^2$, where $m_i$ and $M_w$ stand for the mass of the internal fermion and the mass of the $W$ gauge boson respectively, $F_1$ and $F_2$ are given in the 't Hooft–Feynman gauge by:

$$F_1(q^2 = 0) = Q' \left\{ \begin{array}{l}
\frac{1}{12} \left[ x_i + \frac{13}{12} \right] x_i \\
- \frac{2}{3} \left( 1 - x_i \right)^2 + \frac{5}{6} \left( 1 - x_i \right)^3 \\
+ \frac{1}{2} \left( 1 - x_i \right)^4 \\
\end{array} \right\} \ln x_i,$$

$$F_2(q^2 = 0) = -Q' \left\{ \begin{array}{l}
\frac{1}{41} \left[ \frac{1}{4} + \frac{1}{2} \left( 1 - x_i \right)^2 - \frac{1}{2} \left( 1 - x_i \right)^3 \right] x_i \\
- \frac{3}{2} \left( 1 - x_i \right)^2 \ln x_i \\
+ \left[ \frac{1}{21} + \frac{9}{4} \left( 1 - x_i \right)^2 - \frac{3}{2} \left( 1 - x_i \right)^3 \right] x_i \\
- \frac{3}{2} \left( 1 - x_i \right)^2 \ln x_i \end{array} \right\}. \quad (5)$$

where $Q'$ stands for the electric charge of the internal quark running inside the loop.

Attention should be paid to the "$1/(1 - x_i)\ln x_i$" term standing in the $F_1$ form factor, which would diverge logarithmically as $x_i \rightarrow 0$, that is, if we let the mass of the internal quark "i" tend to zero. As we shall see in the limit: $M_w^2 \gg |q^2| \gg m_i^2 \sim 0$, such term has to be substituted by another one dominated by the piece: $\ln(-q^2/M_w^2)$ [6–8].

The mathematical origin of this singularity can be traced to lie in the $A_4, A_7$ and $A_9$ coefficients, as we shall show below (it is worth to mention that such divergence comes from diagram 1.b).

In the coefficients $A_4$ and $A_7$ one deals (among other pieces of regular behavior in the massless internal fermion limit) with parametric Feynman integrals of the type:

$$I(p^2, (p + q)^2, q^2, m_i^2) = \int_0^1 du_1 \int_0^{1 - u_1} u_2 \frac{f(u_1, u_2)}{\Delta'} du_2,$$

with $f(u_1, u_2) = (u_1 - u_2)(1 + u_2 - u_1)$, and

$$\Delta' = 1 - u_1 + \frac{m_i^2}{M_w^2} u_1 + \frac{q^2}{M_w^2} (u_2 - u_1)(1 + u_2 - u_1) + \frac{2pq}{M_w^2} (1 - u_1)(u_2 - u_1), \quad (6)$$

where we have taken $p^2 = m_i^2 = 0$. Moreover, let us point out henceforth that it is possible to forget about the term "2pq" since it leads, by expanding $1/\Delta'$, to a convergent integral (when $x_i \rightarrow 0$) because of the "$(1 - u_1)$" factor (notice that "$q^2\gamma^\mu - q^\mu\gamma\" in (3) already provides the required second-order in external momenta). Therefore, we write

$$\frac{1}{\Delta'} \equiv 1 - (1 - x_i)u_1 + \beta(u_2 - u_1)(1 + u_2 - u_1) + O(\beta),$$

where we have defined $\beta = q^2/M_w^2$.

Let us observe that, in the limit $x_i \rightarrow 0$, if we drop the $\beta$ term in the denominator, we are confronted to a divergent integral. Hence, under this circumstance, one is compelled to keep such term, perform the integral and then take the limit $x_i \rightarrow 0$. On the other hand, if there is an overall multiplicative $x_i$ factor (as in those parametric integrals coming from diagram 1.c) there is no problem of convergence and it is legitimate to drop the $q^2$-dependent term in $\Delta$. The same happens if the numerator is defined as $f(u_1, u_2) = (1 - u_1)(1 + u_2 - u_1)$. Furthermore,