A NOTE ON PLANAR KINETO-ELASTO-DYNAMICS

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SUMMARIO. L'analisi dinamica di meccanismi flessibili viene generalmente eseguita nell'ipotesi di piccoli spostamenti da una configurazione di riferimento. A questo scopo si definisce comunemente un meccanismo di riferimento composto da membri rigidi. La definizione di tale meccanismo è soggetta ad alcune condizioni restrittive che vengono qui analizzate nel caso di meccanismi articolati piani. La seconda parte di questa nota illustra una forma semplificata di equazioni del moto per meccanismi piani a membri flessibili. Lo spostamento virtuale è approssimato con lo stesso criterio con il quale si approssimano le accelerazioni nel caso che si trascurino i termini secondari. Un paragone tra dati sperimentali e risultati numerici evidenzia come, nel caso in esame, le approssimazioni introdotte corrispondano solamente ad un leggero irrigidimento del modello matematico.

SUMMARY. Dynamic analyses of flexible linkages are generally performed by considering small displacements from a predefined reference configuration. A reference mechanism is commonly defined for this purpose. The definition of a reference mechanism is subjected to some restricting conditions which are analyzed here for planar articulated linkages. A second part of this note documents a simplified form of the equations of motion for flexible planar mechanisms. The virtual displacement is approximated in the same way as accelerations, when secondary terms are neglected. A comparison between experimental findings and results of numerical integrations shows that, in the present case-study, the introduced approximation only corresponds to a light stiffening of the mathematical model.

INTRODUCTION

Kineto-elasto-dynamic approaches, or in other words analyses of flexible multibody systems, have been primarily developed for applications in flexible spacecrafts and flexible mechanisms. Dynamic analyses of mechanisms which take into account member's flexibility began to appear in the literature in the early 70's. A work by Whinfrey [1], which is often considered as one of the earliest works in this field, was published in 1971. Finite element techniques were developed to the well known level in structure mechanics and their diffused use in engineering was the proof of the functionality of these analysis tools: logically the application of these techniques to flexible mechanisms has been one of the most usual approaches to kineto-elasto-dynamics. A recent work by Thompson [2] is an exhaustive review of what has been done over the last fifteen years with finite element techniques in mechanism analysis.

The kineto-elasto-dynamic problem is a non-linear one: small elastic displacements are coupled with the gross motion of the mechanism's members. More solution techniques can be used and fundamental differences between them are a result of the selection of independent coordinates of the system.

Total coordinates can be used to define directly the mechanism actual position in a common global frame, while perturbation coordinates can be adopted to define the actual mechanism small displacements with respect to a predefined reference position. Commonly perturbation coordinates are used in flexible mechanism analysis: in this way the non-linearity is somewhat limited to the gross motion of the reference.

If perturbation coordinates are used a further problem is the choice of the reference from which they can be measured. Frequently the chosen reference is a mechanism composed of rigid links. The definition of a reference mechanism has its great advantage in that sensitivity coefficients can be computed by means of a separated kinematic analysis. But other definitions are also possible for the reference from which the small displacements are considered: this reference is no longer a mechanism, rather it can be a system of unconnected underformed links. These possible reference definitions are a direct consequence of the different techniques which can be used to impose the constraint relations due to the kinematic pairs.

An important problem concerning finite elements applied to kineto-elasto-dynamics is the complexity of the equations of motion. In fact, if the usual perturbation coordinate method is chosen, a considerable amount of coupling terms arises between small displacements and gross motion. This is probably one of the reasons why truly dynamic approaches are so rare in the literature and most examples deal with analyses where the reference motion is computed separately. Some secondary terms were also found to be often negligible, as for example, centrifugal stiffness and gyroscopic damping [2, 3].

The first aim of this paper is to compare some possible reference definitions and to suggest some restrictions which concern the choice of the reference mechanism. A second is to generalize a previously presented truly dynamic approach [4] to the case of multi-d.o.f. planar linkages (modelled
with elements of arbitrary type), and to investigate the effects of certain terms which appear in the global equilibrium equations. To this purpose experimental data [5] are compared with numerical results, obtained using different levels of approximation in the virtual work expression.

REFERENCE DEFINITIONS

As previously mentioned, the reference configuration can be defined as a set of unconnected rigid members, or it can be defined as a reference mechanism made up of rigid links. In the first case constraint relations, due to kinematic pairs, are directly written into the deformed mechanism configuration. In the second case, when a reference mechanism is defined, constraint equations are written into the undeformed configuration and congruence relations for elastic displacements are imposed at the kinematic pairs.

Let us consider a planar four-bar linkage. Fig. 1 shows the case where no reference mechanism is defined. Segments $A_1B_1$, $B_2C_2$ and $C_3D_3$ define members 1, 2 and 3 respectively in their undeformed state. Constraint equations due to revolute pairs are written taking into account small displacements, too. If for example the revolute pair located at $B$ is considered, the constraint equation is:

$$\Delta_{O_1-O_0} + \Delta_{B_1-B_0} + \Delta_{B_2-B_1} + \Delta_{O_2-O_1} = 0.$$  

(1)

In this way, four vector equations can be written for the four-bar linkage including small displacements. But additional conditions have to be satisfied in order to have a set of independent coordinates. Indeed, no rigid-body motion must be possible for each link with respect to its own undeformed configuration. A possible way to overcome this difficulty is to define the members as isostatic structures with respect to their undeformed references: three non-redundant constraint equations are added for each member in a planar analysis. Obviously, there are more possibilities in the definition of the boundary conditions, as for example in Fig. 1, where members 1 and 2 are fixed to their respective references in the midpoints and member 3 is defined as a cantilever at $D$. More generally one can consider the class of planar articulated mechanisms where:

$$n_d = 3m + \sum e_i = 3m + n_e,$$  

(2)

$$c = 3m + 2p,$$  

with $n_d$ being the number of dependent coordinates; $c$ the number of constraint equations; $m$ the number of links excluding link zero which is fixed; $e_i$ the number of elastic coordinates of link $i$; $n_e$ the total number of elastic coordinates (i.e. two or three times the number of nodes of the model); and $p$ the number of revolute and sliding pairs. The number of independent coordinates used in a kineto-elasto-dynamic analysis for this class of mechanisms is:

$$n = n_d - c = n_e - 2p.$$  

(3)

This first approach does not lead to the definition of a reference mechanism: it is typically used when a Lagrange multipliers technique is chosen for the solution of the dynamic problem [6].

However, another more common method of imposing constraint relations is applied: a reference mechanism is defined and congruence relations for elastic displacements are established. This procedure is illustrated in Fig. 2. The constraint equation due to the pair at $B$, which corresponds to (1), is now written as:

$$(O_1 - O_0) + (B_1 - O_1) = 0.$$  

(4)

and the equation:

$$(B_2 - B_1) = (B_2 - B_1)$$  

(5)

insures compatibility of elastic displacements for link 1 and 2 at $B$. In this way, four vector equations can be written, which represent the closure equation of a reference mechanism, and an equal number of congruence equations insures compatibility of elastic displacements. In the more general case of planar articulated mechanisms, the total number of constraint equations using the procedure are:

$$c^* = 2p + 2p.$$  

(6)

The difference between dependent coordinates and constraint equations now becomes:

$$n_d - c^* = (n_e - 2p) + (3m - 2p),$$  

(7)