Carleson's condition. Then \( \mathcal{F} \) is an interpolation sequence for the pair of spaces \( H_p^{(\ell)} \), \( A_p^{(\ell)}(\mathcal{E}) \), \( 1 \leq p \leq \infty \), i.e., \( \mathcal{F} = A_p^{(\ell)}(\mathcal{E}) \).

Moreover, the operator \( \mathcal{S} \) has a linear continuous right inverse: a) for \( 1 \leq p \leq \infty \), this is the mapping \( \mathcal{T}: \mathcal{F} \rightarrow H_p^{(\ell)} \), \( \mathcal{T}(c_n)(z) = \sum_n c_n z^n \), \( z \in \mathbb{D} \); b) for \( 1 \leq p < \infty \), the required operator \( \mathcal{T}, \mathcal{T}: \mathcal{F} \rightarrow H_p^{(\ell)} \) is given by the formula \( \mathcal{T}(c_n)(z) = \sum_n (c_n - c_{n+1}) z^n \).

We note that the functions which realize the interpolation do not have singularities outside the sets \( \{ \alpha_n \} \cup \{ \bar{\alpha}_n \} \). Making use of the theorem of [7], it is easy to see that the interpolation problems considered here and in [3, 4] can be solved in the subclasses of the corresponding classes, consisting of analytic functions with unique singularity at the point \( z=1 \).

**LITERATURE CITED**


A SUPPLEMENT TO THE PAPER "ON LOCALIZABLE FUNCTIONALS IN VECTOR LATTICES"

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It is proved (without the assumption of the continuum hypothesis) that under the condition \( \lim_{t \to 0} \frac{\psi(2t)}{\psi(t)} = 1 \) there exist on the Marcinkiewicz space \( M(\psi) \) abnormal functionals which are not localizable.

Let \( \psi \) be a nondecreasing, continuous, concave function, defined on \([0, 1]\), such that \( \psi(0) = 0 \), \( \psi(1) > 0 \) for \( t > 0 \), \( \lim_{t \to 0} \frac{t}{\psi(t)} = 0 \). By \( \mu \) we denote the Lebesgue measure on \([0, 1]\), \( S \) is the collection of all measurable subsets of \([0, 1]\), \( S \) is the space of all finite real measurable functions on \([0, 1]\) (we identify sets and functions which are equivalent with respect to the measure \( \mu \)). The symbol \( \chi_{E} \) denotes the characteristic function of the set \( E \). For \( x \in S \) by \( x^\ast \) we denote the nonincreasing rearrangement of the function \( |x| \).

The Marcinkiewicz space \( M(\psi) \) consists of all \( x \in \mathbb{S} \), for which \( \| x \| = \lim_{\text{osc} x \to 1} \frac{1}{\psi(h)} \int_0^h x^* d\mu < \infty \). For \( f \in M(\psi)^* \) (\( M(\psi)^* \) is the Banach conjugate of \( M(\psi) \)) and \( E \in \Sigma \) by \( f_E \) we denote the functional on \( M(\psi) \) acting according to the formula \( f_E(x) = f(x \chi_E) \), \( x \in M(\psi) \). The functional \( f \in M(\psi)^* \) is said to abnormal if \( f(x) = 0 \), \( \forall x \in L^\infty[0,1] \). The functional \( f \in M(\psi)^* \) is said to be localizable if \( \forall \varepsilon > 0 \ E \in \Sigma \), for which \( f = f_E \) and \( \mu E < \varepsilon \). Every localizable function is abnormal. In [1] one has proved the following theorem (see [1, Theorem 4]).

**Theorem.** 1. If \( \lim_{t \to 0} \frac{\psi(2t)}{\psi(t)} > 1 \), then every abnormal functional \( f, f \in M(\psi)^* \), is localizable.
2. If \( \lim_{t \to 0} \frac{\psi(2t)}{\psi(t)} = 1 \), then on \( M(\psi) \) there exist abnormal, nonlocalizable functionals. Moreover, in this case there exists an abnormal \( f, f \in M(\psi)^* \), such that \( \| f_E \| = 1 \), \( \forall E \in \Sigma \) with \( \mu E > 0 \).

The second assertion of this theorem has been proved in [1] only under the assumption of the continuum hypothesis. We give here a different proof of the same statement which does not depend on the continuum hypothesis. Assume that the condition \( \lim_{t \to 0} \frac{\psi(2t)}{\psi(t)} = 1 \) holds.

We fix a numerical sequence \( \{a_n\} \) in such a manner that

\[
0 < a_n \leq \frac{1}{n} \quad (n \in \mathbb{N}) \quad \text{and} \quad \lim_{n \to \infty} \frac{\psi(na_n)}{\psi(a_n)} = 1
\]

The existence of such a sequence follows from Lemma 5 of [1]. For \( \tau \in (0,1) \) we set

\[
\psi_\tau(t) = \begin{cases} 
0 & \text{for } 0 \leq t \leq \tau \\
\psi'(t - \tau) & \text{for } \tau < t \leq 1 
\end{cases}
\]

Here \( \psi' = \frac{d\psi}{dt} \). Clearly, \( \psi_\tau \in M(\psi), \| \psi_\tau \| = 1 \). We consider some generalized limit \( \text{Lim} \), defined on the class of all bounded numerical sequences [2]. For \( \tau \in (0,1) \) we construct \( f_\tau \in M(\psi)^* \) according to the formula

\[
f_\tau(x) = \text{Lim} \left( \left\{ \frac{1}{\psi(a_n)} \int_\tau^{\tau+a_n} x(t) dt \right\}_{n \in \mathbb{N}} \right), \quad x \in M(\psi).
\]

Clearly, \( \| f_\tau \| = 1 \), \( f_\tau \) for \( \tau \neq \tau_2 \); \( f_\tau(x_{[0,1]}) = 0 \), since \( \frac{1}{\psi(a_n)} \int_\tau^{\tau+a_n} x_{[0,1]} dt = \frac{a_n}{\psi(a_n)} \int_\tau^{\tau+a_n} \psi(t - \tau) dt = 1 \). From here it follows that \( \| f_\tau \| = 1 \). We show that for any finite collection of mutually distinct points \( \tau_1, \ldots, \tau_m \in (0,1) \) we have the inequality

\[
\| \sum_{k=1}^m f_{\tau_k} \| \leq 1.
\]

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