Probing $b$-quark charged-current chiral structure via polarized-$A_b$ semileptonic decay

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Abstract. $A_b$ semileptonic decay to $A_c e \bar{\nu}_e$ is studied within the heavy quark effective theory, where $A_b$ is moving and polarized along the direction of its motion. Normalized energy distributions of $e$ and $A_c$ are both calculated for $V \pm A$ interactions by taking the $\mathcal{O}(\Lambda/m_c)$ ($\Lambda \equiv m_A - m_c$) corrections and electron $p_t$ cut effects into account. It is shown that the form factor (Isgur-Wise function) effects are significant: The shapes of the distribution curves are thereby considerably changed in comparison with those calculated within the quark model. In case of the electron energy spectrums, the difference between the $V \pm A$ interactions are enhanced, while that in the $A_c$ energy spectrums decreases. On the other hand, the $\mathcal{O}(\Lambda/m_c)$ corrections are found to be negligible in both spectrums.

1 Introduction

A lot of hadron decays have been playing an important role in studying charged-current weak interactions. As a result of their analyses, we know at present the $V - A$ structure of these interactions at least for the light quark sector. This fact has been naturally incorporated into the purely left-handed standard electroweak theory. In treating hadrons, however, we still do not have a well-established general procedure. As a traditional way, we usually express hadron matrix elements in terms of form factors process by process and determine them phenomenologically, or rely on model calculations. For this subject, if we restrict discussions to hadrons containing a single heavy quark ($Q$), a new technique has been developed over the past few years: the heavy quark effective theory (hereafter HQET) [1]. We can thereby express various matrix elements of those hadrons with a small number of form factors. In particular, we need to know only one function, i.e., the so-called Isgur-Wise function in the limit $m_Q \to \infty$, which drastically simplifies treatments of various heavy hadron decays.

Regarding the chiral structure of the weak interactions, on the other hand, Gronau and Wakaizumi have pointed out that the $b$-quark decays have not been probed sufficiently yet to single out $V - A$ current and consequently $V + A$ form cannot be ruled out there [2]. In fact, within the $SU(2)_L \times SU(2)_R \times U(1)$ scheme, they gave a model in which, e.g., $b \to c e \bar{\nu}_e$ occurs purely right-handed. Hou and Wyler also proposed another scheme which allows the same right-handed $b$ decays [3]. To probe this $b$-quark decay structure, Amundson et al. [4] and subsequently Gronau and Wakaizumi [5] have studied $A_b \to A_c e \bar{\nu}_e$ where $A_b$ is running and have polarization along with the direction of its motion. They calculated the final electron energy distribution $\Gamma^{-1} (d\Gamma/dE_{e\bar{\nu}}^{ab})$ ($E_{e\bar{\nu}}^{ab}$ is the electron energy in the $A_b$-running frame) for both the $V - A$ and $V + A$ couplings and found this quantity is quite useful for distinguishing them.

Their work is, however, at the quark level and consequently no form factor effects are included. The heavy quark effective theory is a very interesting and powerful tool to analyze such $A_b$ decays at hadron level. This is because 1) the transition matrix element $\langle A_c | p_{\nu} | A_b \rangle$ can be expressed very concisely as will be seen in Sect. 2, and 2) about 93% of the $b$ quark produced from $Z$-boson decay is polarized and this spin is transferred to that of $A_b$ without being flipped, to a good approximation in this framework. In fact, Mannel and Schuler studied $A_b \to A_c e \bar{\nu}_e$ at the lowest order of the HQET within the standard theory, and found that the form factor effects are significant [6]. (See also the similar work by Körner and Krämer [7]).

Based on such consideration, I applied the HQET to the above-mentioned problem in [8]. That is, I calculated the electron energy spectrums in $A_b \to A_c e \bar{\nu}_e$ by taking into account the $\mathcal{O}(\Lambda/m_c)$ ($\Lambda \equiv m_A - m_c \simeq \mathcal{O}(\Lambda_{QCD})$) corrections and a cut on electron’s $p_t$ (transverse momentum with respect to $A_b$’s momentum) to identify the $A_b$ production [9]. There I also found the form factor effects are very important both in the $V \pm A$ cases. More concretely, these effects drastically change the shapes of these distributions.
and the difference between them is thereby enhanced. In this article, I will show the details of that work and furthermore examine the A~ energy spectrum as well. In the next section, I describe some basic formulation which is necessary for the actual calculations. In Sect. 3, I study the electron energy distribution, and compute that of A_~ in Sect. 4. The final section is devoted to some discussions and conclusion.

2 Formulation

The A_~–A_e transition matrix elements of the vector-current $V^\mu = \bar{c}(1 + \gamma_5) b$ and the axial-vector current $A^\mu = \bar{c}(1 + \gamma_5) \gamma_j b$ at the origin of space-time are generally expressed as

$$\langle A_+(v', s') | V^\mu | A_+(v, s) \rangle = \bar{u}_{A_+}(v', s') \left[ F_1(y) (1 + \gamma_5) + F_2(y) \gamma_5 \right] u_{A_+}(v, s),$$

$$\langle A_+(v', s') | A^\mu | A_+(v, s) \rangle = \bar{u}_{A_+}(v', s') \left[ G_1(y) (1 + \gamma_5) + G_2(y) \gamma_5 \right] u_{A_+}(v, s),$$

where $v(v')$ and $s(s')$ are the velocity and spin of $A_+$ and $A_+$ respectively, and $y = v \cdot v'$. (This $y$ becomes $E_A/m_A$ in the rest frame (CM frame).) In the HQET up to the $O(A/m_A)$ corrections, the above form factors are given by [10]

$$F_1(y) = \frac{1}{2} \left( 1 + \frac{m_A}{m_{A_+}} \right), \quad G_1(y) = \frac{1}{2} \left( 1 + \frac{m_A}{m_{A_+}} \right),$$

$$F_2(y) = \frac{1}{2} \left( 1 + \frac{m_A}{m_{A_+}} \right), \quad G_2(y) = 0,$$

where $c_{QCD}$ is the QCD leading-logarithmic correction factor:

$$c_{QCD} = \left[ \frac{2}{2} \right]^{6/5}.$$

These concise formulas represent one of the interesting points of studying $A_+ \rightarrow A_e e^+ e^-$ in the HQET mentioned in Sect. 1, and $\eta(y)$ is the Isgur-Wise function [1]. What determines the form of $\eta(y)$ is QCD non-perturbative effects, and we have at present no theoretical information on them except for $\eta(1) = 1$ due to the heavy quark symmetry. Here, I assume 1) $\eta(y) = 1 + a_{\text{SW}}(1 - y)$ (linear type), 2) $\eta(y) = 1/[1 - b_{\text{SW}}(1 - y)]$ (pole type), and 3) $\eta(y) = \exp\{c_{\text{SW}}(1 - y)\}$ (exponential type) which were all examined in previous analyses of $B \rightarrow X_u e^+ e^-$ decays [6, 11].

The differential decay width of $A_+$ is given in the covariant normalization as

$$d\Gamma = \frac{1}{2m_{A_+}} \frac{1}{(2\pi)^3} \int d^3 p_e \left| A(A_+ \rightarrow A_e e^+ e^-) \right|^2,$$

where $f = A_+$, $e$ and $\bar{e}$. By assuming that there is no difference in the overall coupling factors between the $V \pm A$ interactions as the model of Gronau and Wakaizumi [2], the decay amplitude $A$ becomes

$$A(A_+ \rightarrow A_e e^-) = G_F \frac{G_F}{\sqrt{2}} V_{cb} L^\mu H_\mu, \quad L^\mu = \bar{u}_{A_+} (1 + a_{\gamma_5}) v_{e}, \quad H_\mu = \langle A_+ | V^\mu + a_q A_+ | A_+ \rangle. \quad (2.6)$$

The parameters $a_{\gamma_5}$ ($= \pm 1$) are for $V \pm A$ couplings. After some calculations, we get

$$\sum \left| L^\mu H_\mu \right|^2 = 32 \left[ (F_1 + a_q a_{\gamma_5} G_1)^2 (p_e \cdot p_A) \right] \left[ (p_{\gamma_5} \cdot p_A) + a_{\gamma_5} m_{A_+} (p_e \cdot \gamma_5) \right] + (F_1 - a_q G_1 F_2) (p_e \cdot p_{A_+}) - a_{\gamma_5} m_{A_+} (F_1^2 - G_1^2) (p_e \cdot p_{A_+}) + (p_{\gamma_5} \cdot p_A, a_q m_{A_+} (p_e \cdot p_{\gamma_5})$$

$$+ m_{A_+} (F_1 + G_1 F_2 (2 p_e \cdot p_{A_+}) (p_{\gamma_5} \cdot p_A) - m_{A_+} (p_e \cdot p_{\gamma_5})$$

$$+ a_{\gamma_5} m_{A_+} [(p_e \cdot p_{A_+}) (p_{\gamma_5} \cdot s) + (p_{\gamma_5} \cdot p_{A_+}) (p_e \cdot s)] \right]. \quad (2.7)$$

Here $\sum$ represents the summation over the spins except for that of $A_+$ and $s$ expresses the usual spin vector. The separation of the CM frame, s is given as $s^\prime = (0, s)$, where $s$ is a unit vector representing the direction of the spin.

3 Electron energy distribution

Mannel and Schuler carried out their calculations in the $A_+$-running frame (Lab frame) from the beginning [6], but I take the way to derive the differential width in the CM frame first and then transform it to the Lab frame as was done in [4]. $d^2\Gamma/(dE_e d\cos \theta_e)$ is calculated from (2.4)–(2.7) in the CM frame as

$$\frac{d^2\Gamma}{dE_e d\cos \theta_e} = \frac{G_F^2 |V_{cb}|^2}{32 \pi^3} \frac{m_{A_+}^3 m_e}{m_A^3} \times \int_{y_{\min}}^{y_{\max}} dy \left[ (F_1 + a_q a_{\gamma_5} G_1)^2 (1 - r^2 + x + 2r y) \right]$$

$$\times \left[ (1 - a_{\gamma_5} P \cos \theta_e) (2 - x - 2r y) \right]$$

$$+ \frac{2}{x} a_{\gamma_5} P \cos \theta_e (1 + r^2 - 2r) \right]$$

$$+ (F_1 - a_q a_{\gamma_5} G_1)^2 (1 + a_{\gamma_5} P \cos \theta_e) x (1 - r^2 - x)$$

$$- 2 (F_1^2 - G_1^2) (1 + a_{\gamma_5} P \cos \theta_e) r (1 + r^2 - 2r)$$

$$- 2 (F_1 + G_1 F_2 (1 - a_{\gamma_5} P \cos \theta_e)$$

$$\times r (1 + r^2 - x (2 - x) - 2r (1 - x)) \right], \quad (3.1)$$

where $x$ and $r$ are defined by using the masses of $A_+$ and $A_e$ and the electron energy as $x = 2E_e/m_{A_+}$ and $r = m_{A_+}/m_{A_e}$. $P$ and $\theta_e$ are respectively the $A_+$ polarization in actual experiments and the angle between the spin quantization axis (parallel to $A_+$'s momentum in the Lab frame $p_{A_+}^{(\text{Lab})}$) and electron’s momentum $p_e$. The electron mass was neglected here. $y_{\max}$ and $y_{\min}$ in the integrals are presented by

$$y_{\max} = \frac{1}{2} (1 + r^2), \quad y_{\min} = \frac{1}{2} (1 - x^2 + r^2).$$

I confirmed that the quark model formula given in [4] is reproduced from (3.1) by setting $c_{QCD} = \eta(y) = 1$ and $\tilde{A} = 0$. By the Lorentz transformation to the Lab frame, the $E_e^{(\text{Lab})}$ distribution is expressed as

$$\frac{d^2\Gamma}{dE_e^{(\text{Lab})}} = \int_{E_{e,\min}}^{E_{e,\max}} dE_e \sqrt{1 - \frac{\beta^2}{\beta E_e}} \frac{d^2\Gamma}{dE_e d\cos \theta_e}$$

(3.3)