Spin Excitations in Light Nuclei: The \( (\vec{p}, \vec{p}') \) Reaction on \( ^{40}\text{Ca} \)

P.M. Boucher\(^1\), B. Castel\(^1\), I.P. Johnstone\(^1\), Y. Okuhara\(^1\), and C. Glashausser\(^2\)

\(^1\) Department of Physics, Queen's University, Kingston, Canada
\(^2\) Department of Physics and Astronomy, Rutgers University, New Jersey, USA

Received November 25, 1988; revised version February 2, 1989

We present a microscopic analysis of the nuclear spin response to proton excitation by examining the specific case of the \( ^{40}\text{Ca}(\vec{p}, \vec{p}') \) reaction. Recent results at LAMPF indicate an interesting trend of a decrease in spin dipole strength and an increase in quadrupole strength when the proton momentum transfer is increased. We show that these effects are consistent with Random Phase Approximation (RPA) results and that, furthermore, the experimental spin response can be used to yield unequivocal information on the strength of the spin-isospin component of the nuclear interaction.

PACS: 24.10; 24.30

1. Introduction

The study of the nuclear spin response continues to attract a great deal of attention in both experimental and theoretical contexts. From the experimental point of view, several recent results of proton inelastic scattering have provided new insight on the problem [1–3]. In particular, the results of recent experiments performed by Glashausser et al. using a 319 MeV polarized proton beam at LAMPF have shown that spin-flip states in \( ^{40}\text{Ca} \) are strongly excited at energies above 10 MeV; moreover, excitations at higher energy, in the neighbourhood of 35 MeV, appear surprisingly enhanced in comparison with single particle model predictions.

At this projectile energy, isospin dependent excitations are known to be excited strongly in the case of spin dependent resonances [4]. Simple shell model considerations suggest that the states at lower energy, in the 10 to 20 MeV region, are of \( \Delta N = 1 \), spin-dipole nature while those at higher energy (30 to 35 MeV) are of \( \Delta N = 2 \), spin-quadrupole type. Both categories of states are pushed toward higher energy by the repulsive spin-isospin interaction. Since many \( 1p-1h \) configurations can contribute to build the spin-dipole and quadrupole resonances, one can expect to learn something about spin dependence of the residual interaction by studying the response of spin-flip resonances in \( ^{40}\text{Ca} \) to an hadronic probe using the Random Phase Approximation (RPA) method.

In Sect. 2 which follows, we describe first the formalism, outlining the reaction analysis before turning to a detailed discussion of the nuclear structure calculation. We then elaborate on a particularly simple and useful procedure to generate the widths of resonance states originating from the coupling to \( 2p-2h \) states and relating them to the imaginary part of an optical potential. Section 3 is devoted to a comparison with recent experiments. There we draw particular attention to the observables which can be instrumental in determining nuclear structure parameters like the Landau spin-isospin parameter \( g_0 \). Finally, we suggest in our conclusion that with the \( g_0 \) value of 0.7 found consistent with earlier nuclear structure calculations, the main features of spin dipole and quadrupole strength in \( ^{40}\text{Ca} \) can be well reproduced through our RPA calculation.

2. Formalism

Our approach will be to describe first the reaction and then the nuclear structure formalisms before discussing the results and comparing them with experiment. The first step is to write the nucleon-nucleon \( t \)-matrix for nucleon-nucleon scattering as [4]

\[
t(q) = t_{\nu}(q) + t_{\gamma}(q) (\sigma^1 \cdot \hat{n}) (\sigma^\rho \cdot \hat{n}) + t_{c}(q) (\sigma^\gamma + \sigma^\rho) \cdot \hat{n} + t_{k}(q) (\sigma^1 \cdot \hat{q}) (\sigma^\rho \cdot \hat{q}) + t_{k}(q) (\sigma^1 \cdot \hat{p}) (\sigma^\rho \cdot \hat{p})
\]  

(1)
where the superscripts p and i denote the projectile and target nucleons respectively. In the above equation, \( t_A(q) \) is used to denote

\[
t_A(q) = t^p_A(q) + t^i_A(q) \tau^p \cdot \tau^i, \text{ etc.}
\]

The choice of coordinate system is as follows:

\[
\hat{\mathbf{q}} = \frac{\mathbf{k}_i - \mathbf{k}_f}{|\mathbf{k}_i - \mathbf{k}_f|}, \quad \hat{\mathbf{n}} = \frac{\mathbf{k}_i \times \mathbf{k}_f}{|\mathbf{k}_i \times \mathbf{k}_f|}, \quad \hat{\mathbf{p}} = \hat{\mathbf{q}} \cdot \hat{\mathbf{n}}
\]

where \( \mathbf{k}_i \) and \( \mathbf{k}_f \) are respectively the initial and final wave vectors. The coefficients of (1) are derived from Love and Franey's nucleon-nucleon t-matrix at \( E_p = 325 \text{ MeV} \) [4]. At this energy, the isospin dependent part of the coefficients in the spin channel is much larger than the isospin independent part. We have also neglected, for simplicity, the spin-orbit interaction term since Love and Franey's work clearly indicates that it has a small effect on the cross section [4]. Thus, only the spin-isospin dependent term of the t-matrix is necessary for our calculation. In the plane wave Born approximation, the spin-flip cross section can be written as

\[
\sigma_{SF} = \frac{m^2}{4\pi^2\hbar^2} \left( \sum_{\lambda} \left| t^\lambda(q) \right|^2 \left( \frac{1}{k} \sum_k \sigma^\lambda \cdot \hat{\mathbf{q}} \sigma^\lambda \cdot \hat{\mathbf{k}} \right)^2 \right) \left( \frac{1}{2} \delta(\omega - \omega_\lambda) \right)
\]

where \( \mathbf{q} \) denotes the momentum transfer and \( \omega \) and \( \omega_\lambda \) the energy transfer and excitation energy of the nuclear state, \( |n\rangle \), respectively.

We now briefly describe the calculation for the nuclear states. The spin-isospin response function was calculated in the RPA using a particle-hole interaction composed of \( \pi^- \) and \( \rho \)-meson exchange components and the Landau parameter \( g_0 \) [5, 6]. The formalism is described comprehensively by Oset et al. [5], to which we refer for more detail (particularly to Appendix C). Only ring diagrams were included in our calculation. The RPA response function, \( \hat{R}_J \), and the unperturbed response function, \( \hat{\Omega}_J \), are \( 2 \times 2 \) matrices in \( (L, J) \) (equal to \( (J, 1) \) in the case of unnatural parity states). The following integral equation has been solved in momentum space by inverting the matrices

\[
\hat{R}_J(q, q'; E) = \hat{\Omega}_J(q, q'; E) + \int \frac{k^2 \, dk}{(2\pi)^3} \hat{R}_J(k, q; E) \cdot \hat{W}(k; E) \hat{R}_J(k, q'; E)
\]

The unperturbed response function can be written as follows:

\[
\hat{\Omega}_J(q, q'; E) = \sum_{\rho h} F^{\rho h}_{J L}(q) \left( \frac{1}{E - E_{\rho h} + i\delta} - \frac{1}{E + E_{\rho h} - i\delta} \right) F^{\rho h*}_{J L}(q)
\]

where

\[
F^{\rho h}(q) = \langle (p h^{-1}) J_L J_L(q) (Y_\rho \otimes \sigma) \tau_1 \tau_2 \rangle.
\]

Finally, the \( p-h \) interaction in momentum space is written as

\[
W_{\rho h} = \left[ V_\rho(E, q) (\sigma(1) \cdot \hat{\mathbf{q}}) (\sigma(2) \cdot \hat{\mathbf{q}}) \right. + V_\rho(E, q) (\sigma(1) \cdot \hat{\mathbf{q}}) (\sigma(2) \cdot \hat{\mathbf{q}}) \tau_1 \tau_2] \]

where the \( \pi \) and \( \rho \) dependent potentials are given by

\[
V_\pi = \frac{f^2_\pi(q)}{m_\pi} \left( \frac{q^2}{E^2 - m^2_\pi - q^2} \right) \quad \text{and} \quad V_\rho = \frac{f^2_\rho(q)}{m_\rho} \left( \frac{q^2}{E^2 - m^2_\rho - q^2} \right)
\]

where \( f_\pi \) and \( f_\rho \) are the familiar coupling constants for \( \pi \) and \( \rho \) meson exchange.

The particle and hole wave functions used for the unperturbed response function were calculated self-consistently using the Hartree-Fock method [7] with the Skyrme interaction SGII [8]. The unbound particle states were determined by diagonalizing the Hartree-Fock Hamiltonian in a large harmonic oscillator space. The RPA calculation included all possible \( p-h \) states up to \( N = 10 \). Once the RPA response function was obtained, we considered the spreading width resulting from the coupling of the RPA states to the \( 2p-2h \) states.

The spreading width in our calculation was determined by the model introduced by Smith and Wambach [9] which relates the coupling to \( 2p-2h \) states with the imaginary part of an optical potential using empirical information from the decay widths of particle and hole excitation. Assuming that the optical potential is symmetric about the Fermi energy, and denoting the particle and hole parts by \( I_p \) and \( I_h \) they obtained the total width,

\[
I^{\text{ST}}(\omega) = \frac{1}{\omega} \int_0^\omega \left[ I_p(e) + I_h(e - \omega) \right] \frac{d\epsilon}{2 C^{\text{ST}}[I_p(e) I_h(e - \omega)]^{1/2}}
\]

Thus, \( I^{\text{ST}} \) was completely specified by single particle data once \( C^{\text{ST}} \) was calculated, thus producing asymmetric widths. The asymmetry is important to give