Determination of the CP violating phase $\gamma$ by a sum over common decay modes to $B_S$ and $\bar{B}_S$

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Abstract. To help the difficult determination of the angle $\gamma$ of the unitarity triangle, Aleksan, Dunietz and Kayser have proposed the modes of the type $K^- D^*_+$, common to $B_s$ and $\bar{B}_s$. We point out that it is possible to gain in statistics by a sum over all modes with ground state mesons in the final state, i.e. $K^- D^*_+, K^* D^*_+, K^- D^*_+, K^* D^*_+$. The delicate point is the relative phase of these different contributions to the dilution factor $D$ of the time dependent asymmetry. Each contribution to $D$ is proportional to a product $F^a F^u f_b f_k$ where $F$ denotes form factors and $f$ decay constants. Within a definite phase convention (i.e. for example the one defined by the heavy quark symmetry in the limit of heavy quarks), lattice calculations do not show any change in sign when extrapolating to light quarks the form factors and decay constants. Then, we can show that all modes contribute constructively to the dilution factor, except the $P$-wave $K^* D^*_+$, which is small. Quark model arguments based on wave function overlaps also confirm this stability in sign. By summing over all these models we find a gain of a factor 6 in statistics relatively to $K^- D^*_+$. The dilution factor for the sum $D_{\text{tot}}$ is remarkably stable for theoretical schemes that are not in very strong conflict with data on $B_\pm$, $B_K$, or extrapolated from semileptonic charm form factors, giving $D_{\text{tot}} > 0.6$, always close to $D(K^- D^*_+)$.  

1 Introduction

Time dependent CP violating asymmetries

\[ A(t) \sim D \text{Im} \left[ \frac{q}{p} \frac{M}{\bar{M}} \right] \sin(AMt) \]  

depends on the physical quantity

\[ \frac{q}{p} \frac{M}{\bar{M}} \]  

which is invariant under phase redefinition of the $|B^0\rangle$, $|\bar{B}^0\rangle$ states. $M$ and $\bar{M}$ are the decay amplitudes of $B^0$ and $\bar{B}^0$ to some common final state $|f\rangle$:

\[ M = \langle f | \mathcal{H}_\omega | B^0 \rangle = \langle f | \mathcal{H}_\omega | \bar{B}^0 \rangle \]  

The mass eigenstates are:

\[ |B_{1,2}\rangle = p|B^0\rangle \pm q|\bar{B}^0\rangle \]  

$D$ is the “dilution factor” that differs from 1 for common final states that are not CP-eigenstates. For the moment, in the expression of the asymmetry (1) we have neglected the possible FSI phases, that we will discuss below. It is obviously important to have a large dilution factor $D$ since the number of needed pairs $B_s$, $\bar{B}_s$ to observe a given asymmetry scales like the $A^{-2}$ or $D^{-2}$.

We will adopt Wolfenstein phase convention and parametrization of CKM matrix [1]. In this convention all CKM matrix elements are real except $V_{ub}$ and $V_{td}$ and it is simple to identify which modes will contribute to the determination of the different angles on the unitarity triangle $\alpha$, $\beta$ and $\gamma$. In the Standard Model we have $|q/p|=1$ to a very good approximation. In Wolfenstein phase convention $|\langle q/p\rangle_B|$ is complex since it depends on $V_{ud}$ while $|\langle q/p\rangle_B|$ is real as it depends on $V_{us}$. On the other hand, the CKM factor of the decay amplitudes is real for $b \to c$ transitions. This gives us three different possibilities for a non-vanishing $\text{Im} [\langle q/p\rangle(M/\bar{M})]$:

1. $b \to u$ transitions of the $B_d - \bar{B}_d$ system, related to the angle $\alpha$;
2. $b \to c$ transitions of the $B_s - \bar{B}_s$ system, related to $\beta$; and
3. $b \to u$ transitions of the $B_s - \bar{B}_s$ system, related to $\gamma$.

Of course, this is only true within Wolfenstein approximation up to $O(\lambda^3)$, and in the tree approximation, since also Penguin diagrams can complicate the picture and pollute the determination of some angles, mostly $\alpha$ and $\gamma$. Examples of the three types of modes, which are CP eigenstates, are respectively: $B_d, \bar{B}_d \to \pi^+ \pi^-$; $B_s, \bar{B}_s \to J/\psi K_s$, and $B_s, \bar{B}_s \to \rho^0 K_s$ [2].
There are several improvements one can think of. First, one can consider modes that can help to cleanly isolate the CP phase we are interested in, avoiding the unwanted phases coming from Penguins. Second, one can try to find non-CP eigenstate common modes to $B^0$ and $B^0$ that, although not so clean as CP eigenstates, can help to increase the statistics [3]. Third, one can consider sums over some channels or semi-inclusive modes that can increase the statistics if they contribute constructively to the asymmetry [4, 5].

In this paper we will be concerned with the angle $\gamma$. Assuming unitarity of the CKM matrix, there are two possible determinations of $\gamma$. If $\alpha$ and $\beta$ are measured through $B_s, B_d$ decays, then $\gamma = \pi - \alpha - \beta$ is in principle known. However, there is an independent check, the possible determination outlines above through $B_s, B_d$ decays, for example the CP eigenstate mode $\rho^0 K_s$ (Fig. 1). This measurement of $\gamma$ will be complicated by the expected quick $B_s - \bar{B}_s$ oscillations, that can wash out the CP asymmetry. Moreover, the mode $\rho^0 K_s$ is expected to have a very small branching ratio ($10^{-6} - 10^{-7}$), since it is not only CKM suppressed by $V_{ub}$, as it is necessary, but it is also color suppressed (a further factor 0.2 in amplitude). This mode is also polluted by Penguin diagrams.

However, there is an alternative way of measuring $\gamma$, namely to consider decay modes of the type $K^- D^+_s$ that are not CP eigenstates but are common to $B_s$ and $\bar{B}_s$, the amplitudes being respectively proportional to $V_{cb} V_{ub}^* \lambda_3$ and $V_{cb} V_{ub} \lambda_3$, both of order $\lambda_3^2$ in terms of the Wolfenstein expansion parameter [6]. This mode is not color suppressed, and one expects a branching ratio of $O(10^{-4})$. Moreover, this mode is not polluted by Penguins. Although we have here the drawback of the dilution factor $D$, both amplitudes $B_s, \bar{B}_s \rightarrow K^- D^+_s$ are of the same order and one can expect a large dilution factor for the CP symmetry, as shown in detail by Aleksan et al. [6].

On the other hand, we have pointed out [4], to help the determination of the angle $\beta$ (for which the popular CP eigenstate $\psi K_s$ is usually proposed), to sum over the common decay modes to $B_s$ and $\bar{B}_s$: $D_+ D^-$ (S-wave, parity violating), $D^+ D^* S + D^* D^-$ (P-wave, parity conserving), $D^* D^*$ (S + D waves, parity violating; P-wave, parity conserving). Each individual mode, although Cabibbo-suppressed has a decay rate of the same order as $\psi K_s$, which is color suppressed. We have shown, making use of the heavy-quark symmetry and transformation properties of the weak interaction under the operator $C P e^{i (\theta / 2) s \gamma}$, where $s \gamma$ is the charm quark spin operator collinear to the momentum, that most of the modes (except the extra change diagram for $D^+ D^* S + D^* D^-$ and the $D^* D^*$ P-wave, which have small amplitudes) contribute constructively to the time dependent asymmetry, giving a total dilution factor very close to one. The gain in statistics relatively to $\psi K_s$ is of the order of 6, although the relative detection efficiency puts $\psi K_s$ and the sum $D^+ D^- + D^+ D^* S + D^* D^- + D^* D^*$ on roughly the same footing.

**2 Calculation of the dilution factor**

In this paper we would like to examine the same possibility in the sum of the modes $K^- D^+_s, K^* D^+_s, K^- D^* S, K^* D^*$ (and also their CP-conjugated), that would help the difficult determination of the angle $\gamma$. This would be even more interesting than for the determination of $\beta$, since we do not have here clean modes like $\psi K_s$ of the latter case. Of course, we do not have here the simple situation of heavy quark symmetry. As we will see below, assuming factorization, each contribution to the dilution factor $D$ is proportional to a product $F^{cb} F^{ab} f_{D_s} f_{K}$ where $F$ denotes form factors and $f$ decay constants. To the heavy-to-heavy meson form factors $F^{cb}$ and heavy meson decay constants $f_{D_s}$, we can apply the heavy-quark symmetry [7], but to the heavy-to-light meson form factors $F^{ab}$ and light meson decay constants $f_K$ we have weaker rigorous results, like the relation between $F^{ab}$ and $F^{bs}$ near zero recoil [8].

The dilution factor $D$ is a physical quantity, independent of the phase convention of the states. It is convenient to work in a precise phase convention, namely the one defined by the heavy quark symmetry in the case of heavy quarks. For light quarks we can adopt the same convention and exploit the empirical fact that the lattice calculations, that extrapolate from heavy masses (at the charm quark, let us say) to light masses, do not find changes of sign for form factor and decay constants. For example, if within the same phase convention $f_K$ would have a different sign than $f_{D_s}$, then lattice calculations would observe the quantity $f_{D_s}/f_K$ to go from 1 to zero and change sign when extrapolating from $P = D$ to $P = K$. This is not what is observed and we conclude that there is stability in sign of the form factors and decay constants when going from heavy mesons to light mesons. This will be crucial to have a reliable estimation of the dilution factor $D$ when summing on the different ground state modes. Moreover, quark model calculations also confirm this stability in sign.

Let us consider the final states common to $B_s(bs)$ and $\bar{B}_s(bs)$

$$| \psi \rangle = | K^- (p) D^+_s ( - p) \rangle$$

$$| K^- (p) D^+_s \rangle (\lambda = 0, - p)$$

$$| K^* (\lambda = 0, p) D^+_s \rangle ( - p)$$

$$| K^* (\lambda = 0, p) D^- ( - p) \rangle$$

$$| K^* (\lambda = \mp, p) D^+_s \rangle (\lambda = \mp, - p)$$

$$| K^* (\lambda = \mp, p) D^- ( - p) \rangle$$

$$| K^* (\lambda = \mp, p) D^+_s \rangle (\lambda = \mp, - p)$$

(5)