QCD corrections to electroweak annihilation decays of superheavy quarkonia

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Abstract. QCD corrections to all the allowed decays of superheavy groundstate quarkonia into electroweak gauge and Higgs bosons are presented. For quick estimates, approximations that reproduce the exact results within less than at worst one percent are also given.

1 Introduction

The search for new quark flavors remains an important and interesting task. The existence of a fourth generation cannot be excluded on the basis of present measurements. The \( \rho \)-parameter only restricts the possible mass-splitting and the limit on the ‘number of neutrinos’ refers to light ones only. In fact, some GUTs even require quarks that do not fit into the standard model scheme.

Because charm as well as bottom have been discovered through hadronic production of their quarkonium bound states \( J/\psi \) and \( \Upsilon \) respectively, some authors [4, 6] examined the prospects for discovering new flavors at future hadron colliders through a similar mechanism. Given favorable circumstances, gluon fusion would be the dominant source for quarkonium production and therefore especially the pseudoscalar ground state \( \eta \) could be produced with sufficient rate. Also the \( ^3S_1 \) state \( \psi \) might be accessible.

For the lighter member of a fourth generation doublet the single quark decay, i.e. the decay of one of the constituents in the quarkonium, is likely to be suppressed due to small intergeneration mixing angles. Thus one may in a first step ignore this channel and only consider the annihilation decays. The latter include some new and distinctive modes which may even offer a way to discover the Higgs boson [4, 5].

Predictions for the decays \( \eta, \psi \to \gamma \gamma, \gamma Z, ZZ, WW, ZH \) have been derived in Born approximation in references [2, 3, 4]. QCD corrections, however, are only partly known. The aim of this work is to fill this gap.

The paper is organized as follows: The calculational method employed can be found in [1] and will be discussed only briefly in the following section 2, together with some general considerations. In section 3 the results of our calculations will be presented. Compact approximations will be given in section 4, a brief summary in section 5.

2 General considerations

Not yet discovered quarks must be heavy and a nonrelativistic treatment of their bound states is adequate. As is well known, in this case the decay width of an S-wave bound state factorizes into a nonperturbative part – the wavefunction at the origin – and a perturbative part which is proportional to the free quark scattering cross section:

\[
\frac{d\Gamma(S \to p_1 + p_2)}{4\pi} = \frac{1}{4m^2} \frac{|R_S(0)|^2}{4\pi} |\mathcal{M}(v = 0)|^2 d_{\text{Lips}} \tag{1}
\]

where \( v \) denotes the relative velocity of the \( q\bar{q} \) system and \( \mathcal{M} \) the free scattering amplitude. The factorization in this form applies for S-states only (for P-states see [1]) and to the order considered in this paper. Relativistic corrections first enter at \( \mathcal{O}(\alpha^2) \), not considered in this work.

There are several ways to calculate the rate: one is to simply compute the spin averaged cross section (with a modified statistical factor \( 1/4 \to 1/(2S+1) \) where \( S \) refers to the spin of the bound state), if only the desired spin configurations can contribute to the sum. This may require care concerning possible couplings. Consider for example the \( \gamma H \)-mode: among the S waves only the spin 1 state with \( J^{PC} = 1^- \) can decay that way, and the approach is straightforward. However, if we switch to the \( ZH \)-mode, both states with \( J^{PC} = 0^- \) and \( J^{PC} = 1^- \) are present in the sum, albeit with unknown relative weight. In this case one may identify the parity violating part \( \propto a \cdot v \) in the amplitude with the \( \eta \) decay, the part \( \propto (v^2 + a^2) \) with \( \psi \). A second way is to project the appropriate amplitudes using the method derived in [1]. Both methods were used to obtain and check the results given below, with the exception of the decays into two Ws. There only the second one was applied because the separation of the various couplings is inconvenient.

The calculation of the wavefunction \( R_S \) requires the knowledge of the QCD potential. To get rid of the dependence on the potential model all widths are normalized to...
\[ \Gamma(\eta \rightarrow \gamma \gamma) = 12\left| R_S(0) \right|^2 \frac{\alpha_s^2 Q^4}{M^2} \]  

(2)

and only the ratios

\[ R_{ab}^X \equiv \frac{\Gamma(X \rightarrow ab)}{\Gamma(\eta \rightarrow \gamma \gamma)} \]  

(3)

are presented.

The zeroth order generic Feynman diagrams responsible for the annihilation decays are shown in Fig. 1. The decay through the virtual photon or Z (Fig. 1(a)) contributes only to the channel \( W^+W^- \), the decay through the virtual Higgs (Fig. 1(b)) is shown only for completeness. It contributes neither to \( \eta \) nor to \( \psi \) decays because of its quantum numbers (this only applies to a standard model Higgs, of course).

First order QCD corrections receive contributions from the diagrams shown in Fig. 2. Their sum is infrared finite. Real gluon emission is forbidden by color conservation. Comparing this result with the calculation of QED corrections, which is practically the same in this order, we could also argue that the coupling of real gluons (photons) to a color (electrically) neutral state vanishes in the static limit, thus providing an infrared finite answer.

However, as expected, all corrections exhibit the Coulomb singularity, e.g.

\[ \Gamma(\eta \rightarrow \gamma \gamma) = \Gamma^{(\text{Born})}(\eta \rightarrow \gamma \gamma) \left( 1 + \frac{\alpha_s C_F}{2\pi} \left( \frac{\pi^2}{\lambda} + \frac{\pi^2}{2} - 10 \right) \right) \]  

(4)

which is universal, proportional to \( \frac{1}{|v|} \) (\( C_F = 4/3 \) is a color factor), and which originates from box- and from s-channel vertex correction diagrams (Figs. 2(a) and (d)). This divergence actually represents part of the Bethe-Salpeter wavefunction and must be dropped since it is already included in the factor \( R_S(0) \). Furthermore, since we are calculating ratios of decay widths, the singularities (formally) cancel anyway. The K-factors for the ratios \( R \), and their nontrivial parts \( \delta k \), which are defined through

\[ K_{ab}^X \equiv \left( \frac{R_{ab}^X}{R_{ab}^{(\text{Born})}} \right)^{\text{corrected}} : \left( \frac{R_{ab}^X}{R_{ab}^{(\text{Born})}} \right)^{\text{Born}} = 1 + \frac{\alpha_s C_F}{2\pi} \cdot \delta k_{ab} \]  

will thus be free from Coulomb singularities. The corrected rate can then be obtained from

\[ \Gamma_{ab}^X \equiv \left( \frac{R_{ab}^X}{R_{ab}^{(\text{Born})}} \right)^{\text{Born}}(\eta \rightarrow \gamma \gamma) \left( 1 + \frac{\alpha_s C_F}{2\pi} \left( \frac{\pi^2}{\lambda} - 10 + \delta k_{ab} \right) \right) \]  

(6)

At this point a comment on the regularization of the Coulomb singularity is appropriate. Two different procedures are possible: one may either start with nonvanishing \( \beta \) and consider the limit \( \beta \rightarrow 0 \) in the end, which obviously requires significantly more effort during the calculation than really needed, in particular for the box diagram. Alternatively one may set \( \beta = 0 \) from the outset and employ a nonvanishing gluon mass \( \lambda \). This second procedure has several advantages: the Coulomb singularity and the infrared divergences are regularized in one step and the special kinematical situation facilitates the calculation (especially of the box diagram) significantly. To connect the two approaches the vertex correction can be investigated. This leads to the substitution rule

\[ \frac{m}{\lambda} \lesssim \frac{\pi}{4|v|} \]

Before the results of the calculations can be presented, the notation must be fixed. Vector and axial vector coupling constants are abbreviated with the help of

\[ v = 2(I_{3L} + I_{3R}) - 4Q \sin^2 \theta_W, \quad a = 2(I_{3L} - I_{3R}), \]

\[ y = 2 \sin 2\theta_W \]

where \( Q \) denotes the quark charge divided by the proton charge, \( I_3 \) (the 3-component of) the weak isospin and \( \theta_W \) the weak mixing angle. The results are applicable to fourth generation quarkonia as well as to more unconventional quarks.