**P-odd effective $Z\gamma\gamma\gamma$ interactions**

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**Abstract.** A general analysis of the lowest-dimensional $P$-odd effective $Z\gamma\gamma\gamma$ interaction is performed. It turns out that there are only two independent gauge invariant dimension-eight effective Lagrangians of such a type, similarly to the $P$-even case discussed earlier. It is shown, that such an effective interaction is naturally induced, at the two-loop level, within an extension of the standard model of electroweak interactions involving two-doublet CP-violating Higgs sector.

1 Introduction

Both theoretical and experimental interest has recently been devoted to the rare decay $Z \rightarrow \gamma\gamma\gamma$ (see [1–5] and the earlier papers [6] for the theory and [7] for the latest experimental limits). Within renormalizable models of electroweak interactions such a process can only occur at one-loop (and higher) level and is completely calculable (i.e. ultraviolet divergences cancel in the corresponding amplitude). The decay rate for $Z \rightarrow \gamma\gamma\gamma$ is very small within the standard model (SM); the relevant branching ratio is about $5.4 \times 10^{-10}$ (see e.g. [1]) and the same seems to be true in some simple extensions of the SM (cf. e.g. [5]). However, it could be significantly enhanced by some particular mechanism when going beyond SM (e.g. such as compositeness [8] or magnetic monopoles [9]) and an observation of this decay mode would therefore be a sensitive probe of various sorts of unconventional "new physics".

In any case, for a model independent treatment of the decay $Z \rightarrow \gamma\gamma\gamma$ it is useful to discuss effective Lagrangians which lead to such a process at the tree level. A part of such an analysis has already been done in [3, 4, 8]. In those papers, the investigation was restricted to the parity-conserving ($P$-even) effective $Z\gamma\gamma\gamma$ interactions (note that such a particular form e.g. corresponds to one-loop graphs in SM—see [4]). The aim of the present paper is to extend the earlier discussion so as to include also the parity-violating ($P$-odd) effective $Z\gamma\gamma\gamma$ interactions.

The paper is organized as follows. In Sect. 2 we first review briefly the earlier results concerning the lowest-dimensional $P$-even effective Lagrangian for the $Z\gamma\gamma\gamma$ interactions and perform then an analogous (though substantially more involved) general analysis of the $P$-odd case. In Sect. 3 an explicit model is shown which leads to a $P$-odd effective $Z\gamma\gamma\gamma$ interactions; the example is provided by an extension of the SM involving two Higgs doublets with $CP$ violation in the Higgs sector. In Sect. 4 the main results are briefly summarized.

2 Lowest-dimensional effective Lagrangians

To begin with, let us recall some well-known general ideas (see e.g. [10, 11]). In the effective Lagrangian approach one generally assumes the existence of an internal mass scale $A$ which is well above the typical energy $E$ of the processes under consideration and the relevant observables in the low energy region $E \ll A$ are expected to be expressed as an expansion in the powers of $E/A$. The scale $A$ characterizes the dynamics beyond the low energy region, it is e.g. the mass of the lightest heavy particle circulating around a closed loop in a Feynman diagram of an underlying theory. The low energy physics is then described by the effective Lagrangian including (in general non-renormalizable) terms with increasing dimension and decreasing importance due to suppression by powers of the inverse mass scale $A$. Therefore, the dominant contribution stems from the lowest-dimensional terms of the effective Lagrangian.

The most general effective Lagrangian for the tree-level interaction $Z\gamma\gamma\gamma$ is constrained by the electromagnetic $U(1)$ gauge invariance, i.e. the only possible building blocks including the electromagnetic field are the components of the field strength $F_{\mu\nu}$. It must contain also the field $Z_{\mu}$: one then obviously needs at least one extra derivative so as to maintain the Lorentz invariance. This implies that the lowest-dimensional effective Lagrangian has dimension eight, i.e. the corresponding coupling constants are proportional to $A^{-4}$, with $A$ being the characteristic mass scale.
Without loss of generality it is possible to attach the extra derivative to the Z field\(^1\) and form the symmetric and antisymmetric combinations\( Z^{\mu\nu}_{\text{eff}} = \partial_{\mu}Z_{\nu} \pm \partial_{\nu}Z_{\mu} \). From these building blocks only two independent Lorentz scalars (i.e. P-even terms) with dimension 8 can be constructed (cf.\([3, 4]\)) if we make use of the (anti) symmetry of the matrices\( F, F^2, F^3 \) and\( Z^{\mu\nu}_{\text{eff}} \) (here we have used an obvious matrix notation for the tensors\( P^\mu, Z^{\mu\nu}_{\text{eff}} \)), namely

\[
X_1 = \langle F^2 \rangle \langle F Z^4 \rangle
\]

and

\[
X_2 = \langle F^3 Z^4 \rangle.
\]

Here and in what follows, the matrix trace over Lorentz indices is denoted as\( \text{tr}(.) \). Thus, the most general lowest-dimensional P-even effective Z\( \gamma\gamma\gamma \) interaction can be described using two independent coupling constants\( G_1 \) and\( G_2 \) with dimension (mass)\(^{-4}\). The corresponding Lagrangian may be written as

\[
\mathcal{L}_{\text{eff}}^{P\text{-even}} = G_1 X_1 + G_2 X_2.
\]

The corresponding decay amplitude is then

\[
A_{\gamma\gamma\gamma\text{Z}\text{-even}} = e^{\mu
u}(k_1)e^{\alpha\beta}(k_2)e^{\gamma\delta}(k_3)c^\gamma(P)A_{\mu\nu\alpha\beta\gamma\delta\text{Z}\text{-even}},
\]

where\( k_i, i = 1, 2, 3 \) are the momenta of the final state photons,\( P \) the momentum of the initial Z,\( e(k_i) \) and\( c(P) \) the corresponding polarization vectors and\( A_{\mu\nu\alpha\beta\gamma\delta\text{Z}\text{-even}} \) is the "polarization tensor" given by Feynman rules derived from the Lagrangian (3). At the tree-level one gets

\[
A_{\mu\nu\alpha\beta\gamma\delta\text{Z}\text{-even}} = 2(2G_1((k_1\cdot k_2)g_{\mu\nu} - k_{1\mu}k_{2\nu})
\times((k_3\cdot P)g_{\mu\nu} - P_{\mu}k_{3\nu})
+ G_2(k_2^2g_{\mu\alpha} - k_{1\beta}g_{\mu\beta})(k_3^2g_{\nu\gamma} - k_{2\gamma}g_{\nu\gamma})
+ (k_3^2g_{\mu\beta} - k_{3\beta}g_{\mu\beta})(P^\mu g_{\alpha\nu} - P_{\mu}g_{\alpha\nu})
+ \text{cycl.(1, 2, 3))}
\]

(5)

From (4), (5) we obtain, after straightforward algebra, the amplitude squared summed over the polarizations of the photons 1, 2 and Z:

\[
\sum_{\text{pol.1,2,3}} |A_{\gamma\gamma\gamma\text{Z}\text{-even}}|^2 = 8(48G_1^2 + 112G_2^2 - 40G_1G_2)
\times(s_{12}^2s_{13}^2 + s_{12}^2s_{23}^2 + s_{13}^2s_{23}^2)
+ 4(90G_1^2 + 25G_2^2 + 88G_1G_2)
\times M_2^2s_{12}s_{13}s_{23} - 4(48G_1^2 + 152G_2^2 + 56G_1G_2)M_2^2
\times((k_1\cdot e(k_3))^2s_{12}^2 + (k_2\cdot e(k_3))^2s_{13}^2
- 2\text{Re}(k_1\cdot e(k_3))(k_2\cdot e(k_3))s_{13}s_{23}),
\]

where\( s_{ij} = k_i\cdot k_j, i, j = 1, 2, 3 \). Note that the\( s_{ij} \) satisfy the constraint (the four-momentum conservation)

\[
s_12 + s_{13} + s_{23} = \frac{1}{2}M_2^2
\]

(7)

and can be expressed in terms of the photon energies\( \omega_i, i = 1, 2, 3 \) in the rest frame of the Z as

\[
s_i = M_2\left(\frac{1}{2}M_2 - \omega_i\right), \quad k \neq i, j
\]

(8)

(the kinematically allowed range for\( \omega_i \) is\( 0 \leq \omega_i \leq M_2/2 \)). Using the gauge condition\( (P\cdot e(k_3)) = 0 \), we have in the rest frame of the Z for the linearly polarized\( \gamma_3 \)

\[
(k_1\cdot e(k_3))^2 = -\frac{1}{4\omega_3^2} \lambda (\omega_1^2, \omega_2^2, \omega_3^2)\cos^2 \phi,
\]

(9)

where\( \lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz \) is the usual Mandelstam function and\( \phi \) is the angle between the polarization 3-vector and the reaction plane. Using the usual Dalitz-plot form of the decay probability, we obtain for the partial width

\[
\Gamma_{\gamma\gamma\gamma\text{Z}\text{-even}} = \frac{1}{(2\pi)^3} \frac{1}{8M_2} \frac{1}{3!} \sum_{\text{pol.1,2,3}} |A_{\gamma\gamma\gamma\text{Z}\text{-even}}|^2
\times \delta \left( \sum_{i=1}^3 \omega_i - M_2 \right) d\omega_1 d\omega_2 d\omega_3 d\Omega d\phi
\]

(10)

Introducing dimensionless variables\( x_i = 2\omega_i/M_2 \), we obtain after the integration over\( dx_1 dx_2 d\Omega \) the energy spectrum of the polarized photon

\[
\frac{d\Gamma_{\gamma\gamma\gamma\text{Z}\text{-even}}}{dx d\phi} = \frac{M_2^9}{552960\pi^4} (G_1^2(1120x^3 - 2080x^4 + 1008x^5
- 240(x^3 - x^4)\cos(2\phi))
+ G_2^2(270x^3 - 490x^4 + 231x^5
- 75(x^3 - x^4)\cos(2\phi))
+ G_1G_2(960x^3 - 1760x^4 + 840x^5
- 280(x^3 - x^4)\cos(2\phi))
)
\]

(11)

Summing this expression over the two perpendicular orientations of the polarization 3-vector (e.g. parallel and perpendicular to the reaction plane) and integrating over\( x \) and\( \phi \) we can reproduce the result of the ref. [3] for the decay rate

\[
\Gamma_{\gamma\gamma\gamma\text{Z\text{-even}}} = \frac{M_2^9}{34560\pi^3} (8G_1^2 + 2G_2^2 + 7G_1G_2).
\]

(12)

\(^2\) Let us note, that in [3] another basis for the effective Lagrangian was used, namely\( \mathcal{L}_1 = F^\mu F_\nu^\alpha Z_\nu Z_\alpha \) and\( \mathcal{L}_2 = F^\mu F_\nu^\alpha F_\nu^\beta P_{\mu\beta} \). Using integration by parts and equations of motion, it is not difficult to obtain the following relations connecting the two bases

\[
\int d^4x X_1 = -8\int d^4x X_2 \quad \text{and} \quad \int d^4x X_2 = 2\int d^4x (X_2 - X_1)
\]

\(^1\) Other possibilities can be obtained using integration by parts and equations of motion.

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