The CVC prediction for the $\tau^- \rightarrow h^- \pi^0 \nu_\tau$ branching ratio

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Abstract. Theoretical predictions for the $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ branching ratio can be made using the measured $e^+e^- \rightarrow \pi^+\pi^-$ cross section and the conserved vector current (CVC) hypothesis. We use a parameterization, that was developed by other authors to study the $\eta$ meson, to fit the $e^+e^- \rightarrow \pi^+\pi^-$ cross section. The parameterization includes both the $\rho$ and $\omega$ resonances and the $\rho'(1450)$ and $\rho''(1700)$ resonances. The result is a good description of the data up to center-of-mass energies 2200 MeV. We compare our result against the fits made by others using the same and different parameterizations of the cross section. Most fits model the cross section except at higher center-of-mass energies where the other fits fail to describe the data. With this parameterization of the cross section, we calculate that the ratio of the $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ branching ratio relative to the $\tau^- \rightarrow e^- \nu_e \bar{\nu}_e$ branching ratio is $R_\pi = 1.37 \pm 0.06$ and that the ratio of the $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ branching ratio relative to the tau lifetime is $L_\pi = (8.40 \pm 0.38) \times 10^{-4}$ fs$^{-1}$. Using a recent set of world averages for the $\tau^- \rightarrow e^- \nu_e \bar{\nu}_e$ branching ratio and lifetime, and including a 0.5% correction for the $\tau^- \rightarrow K^- \pi^0 \nu_\tau$ branching ratio, we predict that the $\tau^- \rightarrow h^- \pi^0 \nu_\tau$ branching ratio is $(25.1 \pm 1.4)\%$. This result is in good agreement with all measurements of the $\tau^- \rightarrow h^- \pi^0 \nu_\tau$ branching ratio.

1 Introduction

There no longer appears to be any inconsistency between the branching ratio of the inclusive single-charged-particle or one-prong $\tau$ decay mode and the sum of all the exclusive one-prong decay modes when the results of an individual experiment is studied [1-3]. OPAL [1] and ALEPH [2] partly resolve the inconsistency with $\tau^- \rightarrow h^- \pi^0 \nu_\tau$ (where $h^{-}$ is either a $\pi^{-}$ or $K^{-}$) branching ratios that are higher than previous measurements [1-10] (see Fig. 1). This recent trend toward higher $\tau^- \rightarrow h^- \pi^0 \nu_\tau$ branching ratios lead us to review the current theoretical predictions.

Theoretical predictions for the $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ branching ratio [11-14] can be made using the measured $e^+e^- \rightarrow \pi^+\pi^-$ cross section and the conserved vector current (CVC) hypothesis [15]. The $\tau^- \rightarrow h^- \pi^0 \nu_\tau$ branching ratio is obtained by adding the $\tau^- \rightarrow K^- \pi^0 \nu_\tau$ branching ratio of $(0.5 \pm 0.1)\%$ [16] to the $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ branching ratio. The CVC hypothesis relates the decay width, $I(\tau^- \rightarrow \pi^- \pi^0 \nu_\tau)$, to the isospin 1 component of the $e^+e^- \rightarrow \pi^+\pi^-$ cross section. Consequently, the cross section must be fit with a model to determine the isospin 1 part.

A number of parameterizations have been used to fit the $e^+e^- \rightarrow \pi^+\pi^-$ cross section. All include the $\rho(770)$ and $\omega(783)$ resonances. Where they differ is in their choice for the shape of the resonances, and in their treatment of the energy region above the $\rho$ and $\omega$, where additional resonances are observed. The fits presented in this paper use the parameterization developed by Benayoun et al. [17]. The form of the parameterization is given in [17] but we briefly describe it in Appendix A.1. Kuhn and Santamaria have also fit the cross section with a number of parameterizations [13], and in Appendix A.2 we discuss one of their parameterizations.

The prediction of the $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ branching ratio requires a good model of the $e^+e^- \rightarrow \pi^+\pi^-$ cross section. We begin by fitting the energy region around the $\rho$ resonance to estimate the parameters of the $\rho$ and $\omega$ resonances, and to compare with the results of other fits. The data above the $\rho$ resonance cannot be described unless additional resonances are included [18]. We add the $\rho'(1450)$ and $\rho''(1700)$ to the parameterization using parameters close to those given in the Particle Data Group (PDG) Summary [19].

The decay width, $I(\tau^- \rightarrow \pi^- \pi^0 \nu_\tau)$, can be calculated from the cross section using the CVC hypothesis. To relate our predicted value for the decay width to experimental measurements, we need to re-express it as a branching ratio. The $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ branching ratio can be calculated from the $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decay width in two ways. In the first approach, we define the quantity $R_\pi$, to be ratio of the $\tau^- \rightarrow \pi^0 \nu_\tau$ decay width to the $\tau^- \rightarrow e^- \nu_e \bar{\nu}_e$ decay width. This follows the notation of...
measurements of these quantities are not quite consistent.

The decay width, \( \Gamma(\tau^- \rightarrow \pi^- \pi^0 \nu_\tau) \), can be obtained from the width, \( \Gamma(\tau^- \rightarrow \pi^- \pi^0 \nu_\tau) \), and can be written as

\[
\Gamma(\tau^- \rightarrow \pi^- \pi^0 \nu_\tau) = \frac{B_{e^- \pi^0}}{\tau_{\pi^-}}.
\]

In the second approach, we define the quantity, \( L_{\rho} \), to be equal to the ratio of the measured \( \tau^- \rightarrow \pi^- \pi^0 \nu_\tau \) branching ratio and \( \tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau \) branching ratio, \( B_{e^- \pi^0} \). The ratio, \( L_{\rho} \), can be written as

\[
L_{\rho} = \frac{\Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)}{\Gamma(\tau^- \rightarrow \pi^- \pi^0 \nu_\tau)} = \frac{B_{e^- \pi^0}}{B_{e^- \nu}}.
\]

These two methods of calculating the \( \tau^- \rightarrow \pi^- \pi^0 \nu_\tau \) branching ratio should give the same result. Lepton universality predicts a direct relationship between the \( \tau \) lifetime and the \( \tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau \) branching ratio. However, the measurements of these quantities are not quite consistent with the standard model prediction, though the significance is diminishing with more precise data (for a recent review, see [20]). Still, this inconsistency leads to an additional uncertainty in the predicted \( \tau^- \rightarrow \pi^- \pi^0 \nu_\tau \) branching ratio.

In this paper we briefly describe how the branching ratios are extracted from the fits to the \( e^+ e^- \rightarrow \pi^+ \pi^- \) cross section. The results of the fits are then presented and the \( \tau^- \rightarrow \pi^- \pi^0 \nu_\tau \) decay width is determined. The ratio, \( R_{\rho} \), of the \( \tau^- \rightarrow \pi^- \pi^0 \nu_\tau \) branching ratio with respect to the \( \tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau \) branching ratio and the ratio, \( L_{\rho} \), of the \( \tau^- \rightarrow \pi^- \pi^0 \nu_\tau \) branching ratio with respect to the \( \tau \) lifetime are calculated. Using world average values for the \( \tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau \) branching ratio and \( \tau \) lifetime, we make a prediction for the \( \tau^- \rightarrow h^- \pi^0 \nu_\tau \) branching ratio and compare this result against experimental measurements.

2 Conserved vector current hypothesis

The decay of the \( \tau^- \rightarrow \pi^- \pi^0 \nu_\tau \) proceeds through the hadronic weak vector current. The coupling strength of the weak vector current to \( \pi^- \pi^0 \) is related to the coupling strength of the electromagnetic vector current to \( \pi^- \pi^- \) by the CVC hypothesis [15]. The electromagnetic coupling can be calculated from the measured \( e^+ e^- \rightarrow \pi^+ \pi^- \) cross section and without including radiative corrections, the decay width, \( \Gamma(\tau^- \rightarrow \pi^- \pi^0 \nu_\tau) \), is related to the cross section by [11]

\[
\Gamma(\tau^- \rightarrow \pi^- \pi^0 \nu_\tau) = \frac{G_F^2 \cos 2\theta_C m_t^2}{128\pi^2 m_\tau^2} \int Q^2 dQ^2 (m_\tau^2 - Q^2)^2 \times (m_\tau^2 + 2Q^2) \sigma_{e^+ e^- \rightarrow \pi^+ \pi^-}(Q^2) \tag{3}
\]

where \( G_F \) is the Fermi coupling constant, \( z \) is the fine structure constant, \( m_t \) is the tau mass, \( \sqrt{Q^2} \) is the center-of-mass energy of the electron-positron pair, \( \cos \theta_C \) is the cosine of the Cabibbo angle, and \( \sigma_{e^+ e^- \rightarrow \pi^+ \pi^-}(Q^2) \) is the isospin 1 part of the \( e^+ e^- \rightarrow \pi^+ \pi^- \) cross section.

As indicated in the first section, the \( \tau^- \rightarrow \pi^- \pi^0 \nu_\tau \) branching ratio can be obtained from the width, \( \Gamma(\tau^- \rightarrow \pi^- \pi^0 \nu_\tau) \), in two ways. The quantity, \( R_{\rho} \), is defined as the ratio of the \( \tau^- \rightarrow \pi^- \pi^0 \nu_\tau \) width to the \( \tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau \) width and can be written as

\[
R_{\rho} = \frac{\Gamma(\tau^- \rightarrow \pi^- \pi^0 \nu_\tau)}{\Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)} = \frac{B_{e^- \pi^0}}{B_{e^- \nu}} = \frac{3\cos 2\theta_C m_t^2}{2\pi^2 m_\tau^2} \int Q^2 dQ^2 (m_\tau^2 - Q^2)^2 \times (m_\tau^2 + 2Q^2) \sigma_{e^+ e^- \rightarrow \pi^+ \pi^-}(Q^2) \tag{4}
\]

where \( \Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau) = G_F^2 m_\tau^2 /192\pi^3 \). This is equivalent to the ratio of the respective branching ratios (see equation 1). Alternatively, the quantity, \( L_{\rho} \), is defined to be equal to the width, \( \Gamma(\tau^- \rightarrow \pi^- \pi^0 \nu_\tau) \), and re-expressed in terms of the ratio of the \( \tau^- \rightarrow \pi^- \pi^0 \nu_\tau \) branching ratio to \( \tau \) lifetime (see (2)).

Higher order corrections have been calculated for both the \( \tau^- \rightarrow \pi^- \pi^0 \nu_\tau \) and \( \tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau \) channels. Inclusion of the electroweak radiative corrections as predicted by Marciano and Sirlin [21] reduce \( \Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau) \) by a factor of 0.996. In addition, Marciano [14] argues that there are short-distance loop enhancements which increase the widths of hadronic tau decays by approximately 1.019 and that there is an uncertainty in the order \( \alpha \) corrections (beyond the short-distance effects) of approximately 2%. The terms, \( R_{\rho} \) and \( L_{\rho} \), will be corrected for these higher order corrections.

3 Fits to the \( e^+ e^- \rightarrow \pi^+ \pi^- \) cross section

In this section we present our fits to the \( e^+ e^- \rightarrow \pi^+ \pi^- \) cross section using the parametrization described in Appendix A.1. The data is shown in Figures 2 and 3, and is from the CMD [22], OLYA [23] and DM1 [24] experiments (the data is also tabulated in [25]), and the DM2 experiment [18]. Two fits, the first in the 600–1000 MeV region, and the second in the 600–2200 MeV region, were made to the data. The first fit gives a good estimate of the parameters of the \( \rho \) and \( \omega \) resonances (as well as \( R_{\rho} \)).