\( \pi - A_1 \) electromagnetic form factors and light-cone QCD sum rules

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Received: 19 April 1994

Abstract. Electromagnetic form factors of the transition \( \pi + \gamma_{\text{virt.}} \rightarrow A_1 \) are calculated by QCD sum rules technique with the description of the pion in terms of the set of wave functions of increasing twist. Obtained results are compared with standard QCD sum rule calculations.

Recently it was suggested to study pion form factor at not very large momentum transfers by light-cone QCD sum rules [1] which combine the description of pion in terms of the set of wave functions of increasing twist with the technique of QCD sum rules [2]. This approach gives a possibility to calculate a contribution of the so-called Feynman mechanism to the pion form factor, in which large momentum transfer selects the configuration, in which one parton carries almost all the momentum of the hadron. In this paper [1] it was shown that at least up to the momentum transfers of order 10 GeV^2 this mechanism remains important and it is possible to describe pion form factor in this region by Feynman mechanism only. The second mechanism of large momentum transfer is the hard rescattering mechanism, in which large momentum transfer selects configurations with a small transfers size in which the momentum fraction carried by interacting quark remains an average one. The hard rescattering mechanism involves a hard gluon exchange, and can be written in the factorized form [3–5].

There exists a number of arguments in favour of that at large but finite momentum transfer \( Q^2 \sim 1 - 10 \text{ GeV}^2 \) Feynman mechanism is dominant. Using QCD sum rule approach [2] it was found [6,7] that pion form factor at \( Q^2 \sim 1 - 3 \text{ GeV}^2 \) is saturated by Feynman type contribution. However, this method cannot be used for higher momentum transfer due to increasing contribution of higher operator product expansion terms at large \( Q^2 \). Nevertheless in [8,9] using a concept of nonlocal condensates it was obtained indications that Feynman type contribution is dominate at least up to \( Q^2 \sim 10 \text{ GeV}^2 \). At the same time an attempt to describe the data of pion form factor at \( Q^2 \geq 3 \text{ GeV}^2 \) by the contribution of hard scattering only leads to conclusion that the low energy pion wave function is very far from its asymptotic form [10] and there was proposed a model for pion wave function which has a peculiar "humped" profile which corresponds that the most probable pion configuration is the case when one of the quark carry almost all pion momentum. But in [11] it was pointed that the wave function of such type corresponds to, respectively, soft gluon exchange even at \( Q^2 \sim 10 \text{ GeV}^2 \).

Here we discuss Feynman type mechanism for \( \pi \rightarrow A_1 \) electromagnetic pion form factors. The first calculation of this amplitude was made in [12] by using operator product expansion for three-points correlator in vacuum.

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Let us consider the correlator:

\[ T_{\mu \nu}(p, q) = \int e^{ipx} d^4x \langle 0 | T \{ j^\mu(x), j^\nu(0) \} | \pi(k) \rangle, \]

where \( j^\mu = d^\mu uT su \) and \( j^\nu = 2@^\nu u - 2uid \) is the electromagnetic current, \( k \) is momentum of pion. This correlator was used in [1] to study pion form factor.

Leading twist operator gives the following contribution to \( T_{\mu \nu} \):

\[ T_{\mu \nu}(p, q) = f_n \int_0^1 du \frac{\varphi_\mu(u)}{(1 - u)p^2 + uq^2} \left( \frac{1}{2}(p^2 - q^2)g_{\mu \nu} \right. \]

\[ - 2(1 - u)p_\mu p_\nu + (1 - 2u)(p_\mu q_\nu + q_\mu p_\nu) + 2uq_\mu q_\nu \bigg), \]

where \( \varphi_\mu(u) \) is twist-2 pion wave function. We use the following definition for twists \(-2\) and \(-4\) two-particle
wave functions of pion:

\[ \langle 0 | \bar{d}(0) \gamma_5 \gamma_5 u(x) | \pi(k) \rangle = i f_{\pi} k_{\mu} \int_0^1 e^{-ikx}(\phi(u) + x^2 g_1(u)) \]

\[ + \mathcal{O}(x^4) \, du + f_{\pi} \left( x_{\mu} - \frac{k_{\mu}}{k x} x^2 \right) \int_0^1 e^{-ikx} g_2(u) \, du + \ldots \]  

(3)

Here \( g_1 \) and \( g_2 \) are twist-4 pion wave functions.

In this paper we use different models for pion wave function. The first one is asymptotical wave function. In [3] it was shown that at asymptotically large \( Q^2 \) the pion wave function of leading twist has the following form:

\[ \phi^{(as)}(u) = 6u(1 - u). \]  

(4)

The attempt to describe pion form factor at \( Q^2 \geq 3 \text{ GeV}^2 \) by hard rescattering mechanism only leads to a conclusion that the form of the wave function is much different from asymptotical one and it was suggested to use the following model for pion twist-2 wave function (see [10]):

\[ \phi^{(tw)}(u, \mu \sim 500 \text{ MeV}) = 30u(1 - u)(2u - 1)^2. \]  

(5)

Also the third model pion wave function is

\[ \phi^{(BF)}(u) = 6u(1 - u) \left( 1 + A_2 \frac{3}{2} \left( 5(2u - 1)^2 - 1 \right) \right. \]

\[ \left. + A_4 \frac{15}{8} \left[ 21(2u - 1)^4 - 14(2u - 1)^2 + 1 \right] \right), \]

(6)

which was proposed by Braun and Filyanov in [13] at low normalization point \( \mu \simeq 0.5 \text{ GeV} \), with coefficients \( A_2 = \frac{3}{2} \) and \( A_4 = 0.43 \) which is in agreement with QCD sum rules for first moments of pion wave function and provides that at a middle point \( u = 0.5 \)

\[ \phi_{\alpha}(0.5) \approx 1.2. \]  

(7)

This value is with experimental values of various hadronic coupling constants calculated by light-cone sum rules approach (see [13]).

It is easy to check that (2) satisfies to Ward identities:

\[ p_{\mu} F_{\nu\mu} = - f_{\pi}(p - q)_\mu, \quad q_{\mu} F_{\nu\mu} = - f_{\pi}(p - q)_\mu. \]  

(8)

Twist-4 quark wave functions \( g_1(u) \) and \( g_2(u) \) give the following contribution into the correlator (1):

\[ f_{\pi} \int_0^1 du \left( \frac{1}{(1 - u)p^2 + uq^2)^2} \left[ 4g_1(u) \left( \frac{1}{2}(p^2 - q^2) \right) g_{\mu\nu} \right. \right. \]

\[ - 2(1 - u)p_{\mu} p_{\nu} + (p_{\mu} q_{\nu} + q_{\mu} p_{\nu})(1 - 2u) + 2uq_{\mu} q_{\nu} \]

\[ - 4g_2(u)(1 - u)^2 p_{\mu} p_{\nu} + (1 - u)(p_{\mu} q_{\nu} + q_{\mu} p_{\nu}) \]

\[ + u^2 q_{\mu} q_{\nu} - 4g_2(2u)(1 - 2u)p_{\mu} p_{\nu} - (1 - 2u) \]

\[ \times (p_{\mu} q_{\nu} + q_{\mu} p_{\nu}) - 2uq_{\mu} q_{\nu} \right) \left. \right] + \frac{2g_2(u)}{(1 - u)p^2 + uq^2) g_{\mu\nu}, \]

(9)

where \( g_2(u) = - \int_0^u g_2(v) \, dv \).

There are also quark-gluon twist-4 operators:

\[ \langle 0 | \bar{d}(0) \gamma_5 \gamma_5 u G_{\mu\nu}(v(x)) u(x) | \pi(k) \rangle \]

\[ = k_{\mu}(k_{\alpha} x_{\rho} - x_{\alpha} k_{\rho}) \frac{1}{\pi} f_{\pi} \int \right. \left. e^{-ikx(x_1 + ox_3)} Dz \Phi_{\|}(x_1) \right. \]

\[ + \left( k_{\mu}(g_{\mu\nu} - k_{\mu} x_{\nu}) - k_{\nu}(g_{\mu\nu} - k_{\mu} x_{\nu}) \right) \]

\[ \times f_{\pi} \int e^{-ikx(x_1 + ox_3)} Dz \Phi_{\perp}(x_1), \]

(10)

\[ \langle 0 | \bar{d}(0) \gamma_5 \gamma_5 \bar{d} G_{\mu
u}(v(x)) u(x) | \pi(k) \rangle \]

\[ = k_{\mu}(k_{\alpha} x_{\rho} - x_{\alpha} k_{\rho}) \frac{1}{\pi} f_{\pi} \int \right. \left. e^{-ikx(x_1 + ox_3)} Dz \Phi_{\|}(x_1) \right. \]

\[ + \left( k_{\mu}(g_{\mu\nu} - k_{\mu} x_{\nu}) - k_{\nu}(g_{\mu\nu} - k_{\mu} x_{\nu}) \right) \]

\[ \times f_{\pi} \int e^{-ikx(x_1 + ox_3)} Dz \Phi_{\perp}(x_1), \]

(11)

where \( \bar{G}_{\mu\nu} = \frac{1}{2} \varepsilon_{\alpha\beta\gamma\delta} G_{\mu\nu}, \) \( Dz = dz_1 dz_2 dx_3 \delta(1 - x_1 - x_2 - x_3). \)

The contribution of these operators (10,11) is

\[ f_{\pi} \int_0^1 du \left( \frac{1}{((1 - u)p^2 + uq^2)^2} \right) \left[ (2(1 - u)p_{\mu} p_{\nu} \right. \]

\[ - (1 - 2u)(p_{\mu} q_{\nu} + q_{\mu} p_{\nu}) - 2uq_{\mu} q_{\nu}) A(u) \]

\[ - \frac{1}{2} g_{\mu\nu}(p^2 - q^2) B(u) + 2(p_{\mu} q_{\nu} - q_{\mu} p_{\nu}) C(u) \right) \],

(12)

where

\[ A(u) = \int_0^u \int \left[ \frac{1 - z_1}{u - z_1} \frac{d z_3}{z_3} \left( \Phi_{\|}(x_1) + 2\Phi_{\perp}(x_1) \right) \right] \]

\[ + \left( 1 - 2 \frac{u - z_1}{z_3} \right) \left( \Phi_{\|}(x_1) + 2\Phi_{\perp}(x_1) \right) \],

(13)

\[ B(u) = \int_0^u \int \left[ \frac{1 - z_1}{u - z_1} \frac{d z_3}{z_3} \left( \Phi_{\|}(x_1) \right) \right] \]

\[ + \left( 1 - 2 \frac{u - z_1}{z_3} \right) \left( \Phi_{\|}(x_1) \right), \]

(14)

\[ C(u) = \int_0^u \int \left[ \frac{1 - z_1}{u - z_1} \frac{d z_3}{z_3} \left( \Phi_{\|}(x_1) \right) \right] \]

\[ + \left( 1 - 2 \frac{u - z_1}{z_3} \right) \left( \Phi_{\perp}(x_1) \right). \]

(15)

A systematic study of the higher twist wave functions has been done in the paper [13]. The set of wave functions suggested in the paper includes contributions of operators with lowest conformal spin and also the corrections corresponding to the operators with next-to-leading conformal spin, which numerical values were estimated by the QCD sum rule method. This set is (hereafter