Pair production of squarks in $ep$-collisions

K.J.F. Gaemers and M.M.J.F. Janssen
NIKHEF, P.O. Box 41882, 1009 DB Amsterdam, The Netherlands

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Abstract. The production of squarks at $ep$ colliders is studied by comparing the Weizsäcker-Williams approximation and an exact tree level calculation. We consider the process $ep \rightarrow e\bar{q}\bar{q}X \rightarrow eq\bar{q}\bar{q}X$. For this calculation, on the amplitude level, the Feynman diagrams can be split into sub-diagrams. This splitting has great advantages for numerical calculations. Using these techniques the dependence of the cross section on the mass of the squark is computed. We also present various kinematical distributions. An elegant method for calculating tree amplitudes with external fermions of arbitrary mass and spin is given for completeness.

1 Introduction

We consider a supersymmetric extension of the Standard Model. In such a model there are the familiar leptons, quarks and gauge bosons, together with the new supersymmetric partners, the spin 0 squarks and sleptons and the spin $\frac{1}{2}$ gauginos. The model is described e.g. in [1]. It is of course important to see whether nature is described by models of this type. One way to study this question is to see whether the supersymmetric partners can be produced at high-energy (future) colliders. In this paper we will consider the production of a squark together with an anti-squark.

The main reasons for examining this process again is, that in the literature [6] and [7], there is some confusion about the results in the WW-approximation. In addition to a recalculation of the WW-approximation, we present a complete tree level calculation without additional approximations. The advantages of an exact calculation is that it allows to study in detail various distributions of the final state electron as well as for the final state squarks and their decay products.

In this paper, complete helicity amplitudes for the process $eg \rightarrow e\bar{q}\bar{q}$ are given for arbitrary squark masses. We also keep a finite electron mass. This is important because it provides for a physical cut-off on the momentum transfer.

The full amplitude is the sum of three diagrams in Fig. 1a, b and c. It turns out that two of the three diagrams can be factorized and can be interpreted as a $2 \rightarrow 2$ times a $1 \rightarrow 2$ process. Further, the third diagram, the one with the four point vertex, can be simplified in such a way that it can be interpreted as a $1 \rightarrow 2$ process. The advantage of the helicity amplitudes is, that a rather complicated diagram can be reduced into simpler processes.

This paper is organized as follows. In Sect. 2 the process in the WW-approximation is considered. In Sect. 3 the full matrix element is written down and it is shown that it can be reduced into simple sub-processes. In Sect. 4 the exact expression for the process $ep \rightarrow e\bar{q}\bar{q}X$ is presented. In the last section numerical results are given and we discuss the choices we made for the gluon distribution, strong coupling constant etc. In the appendix the method of evaluating tree amplitudes in the Weyl basis is described.

2 The WW-approximation for $e^- g \rightarrow e^- \bar{q}\bar{q}$

In the Weizsäcker-Williams approximation the production of squark anti-squark pairs is reduced to photon gluon fusion. The Feynman diagrams for this process
Fig. 2a–c. The Feynman diagrams used in the Weizsäcker-Williams-approximation, $\gamma g \rightarrow q\bar{q}$

are depicted in Fig. 2. The total cross section for $\gamma g \rightarrow q\bar{q}$ is given by

$$\sigma(s) = \frac{\pi x Q_s^2 x_s(Q^2)}{s} \left[ 2(2-\beta^2) \beta - (1-\beta^4) \ln \frac{1+\beta}{1-\beta} \right]$$

(1)

where

$$s=(p_1+p_2)^2$$

(2)

$$\beta=(1-4m_q^2/s)^{1/2}$$

(3)

$$Q^2=s/2$$

(4)

and $\alpha$ is the fine structure constant, $Q_s$ is the charge of the squark and $x_s(Q^2)$ is the strong running coupling constant. This cross section is the incoherent sum over $q_L$ and $q_R$ with $m_{q_L}=m_{q_R}$.

We can use this result for the total cross section in $ep$ collisions,

$$\sigma(e^-p\rightarrow e^-q\bar{q}X) = \int_{x_{\text{min}}} \int_{z_{\text{min}}} f_{\gamma/e}(z, E)$$

(5)

$$G(x, Q^2)h(xxs)$$

with

$$z_{\text{min}}=(m_q+m_{\bar{q}})^2/xs$$

(6)

$$x_{\text{min}}=(m_q+m_{\bar{q}})^2/s$$

(7)

and $s$ is the center of mass energy squared of the $ep$ system.

The function

$$f_{\gamma/e}(z, E)=\frac{x}{\pi} \frac{1+(1-z)^2}{z} \frac{E(1-z)}{m_e z}$$

(8)

describes the equivalent photon distribution in $z=p_e/p$ for $E=\sqrt{s}/2$. $G(x, Q^2)$ denotes the gluon distribution in the proton. In the literature [6, 7] two-photon distributions have been used for the production of a squark anti-squark pair. We have checked that cross sections based on these different distributions vary by less than a few percent. We will discuss the results in Sect. 5.

3 The matrix element

In Fig. 1a, b, c, the diagrams which at tree level give the main contribution to the cross section for the process $e^- g \rightarrow e^- q\bar{q}$, are shown. The corresponding amplitudes for the three diagrams in Fig. 1a, b and c are denoted by $M_1$, $M_2$ and $M_3$ respectively, where $M_1$, $M_2$ and $M_3$ still depend on the spins of the external particles: $\lambda_1, \lambda_2, \lambda_3 = \pm$. We assume throughout that the left- and right-squarks have the same mass and do not mix.

In the expressions below there is already averaged over all colours of the squarks. From our choice of normalization of the $SU(3)$ matrices, we obtain a colour factor, $C_{\text{colour}}=1/\sqrt{2}$, which must be multiplied at the end with the matrix element. For definiteness we first consider squarks of the left type.

The expressions for $M_1$, $M_2$ and $M_3$ are,

$$M_1(\lambda_1, \lambda_2, \lambda_3) = M_{1a}(\lambda_2, \lambda_3) \frac{i}{p_x^2 - M_q^2} \cdot M_{1b}(\lambda_4)$$

(9)

where

$$M_{1a} = \sum_{\pm} i(g_{eL})_{\lambda_2} \bar{u}_{\lambda_5} p_{uL} u_{pL} iD_{\mu} i g_{\gamma} (p_4 + p_3)$$

(10)

$$M_{1b} = i g_{\gamma} (p_x - p_3) \delta^\mu(p_1)$$

(11)

for $p_x = p_1 - p_3$, $p_x = p_1 - p_3$ and

$$M_2(\lambda_1, \lambda_2, \lambda_3) = M_{2a}(\lambda_2, \lambda_3) \frac{i}{p_x^2 - M_q^2} \cdot M_{2b}(\lambda_4)$$

(12)

where

$$M_{2a} = \sum_{\pm} i(g_{eL})_{\lambda_2} \bar{u}_{\lambda_5} \gamma^\mu p_{uL} iD_{\mu} i g_{\gamma} (-p_3 - p_4)$$

(13)

$$M_{2b} = i g_{\gamma} (p_4 - p_3) \delta^\mu(p_1)$$

(14)

for $p_x = p_1 - p_4$ and

$$M_3(\lambda_1, \lambda_2, \lambda_3) = \sum_{\pm} i(g_{eL})_{\lambda_2} \bar{u}_{\lambda_5} \gamma^\mu D_{\mu} 2 i g_{\gamma} g_{\gamma} \delta^\mu(p_1)$$

(15)

with,

$$D_{\mu \nu} = (-g_{\mu\nu} + k_{\mu} k_{\nu}/M^2)/(k^2 - M^2)$$

for a $Z$-boson

$$D_{\mu \nu} = -g_{\mu\nu}/k^2$$

for a photon

and $k = p_2 - p_3$. Further, $e_{\gamma}(p_1)$ is the gluon polarization vector and $P_{\gamma}$ is the chiral projection operator. The $(g_{eL})_{\lambda_2}$ and $g_{\gamma}$ are the weak coupling constants for leptons and quarks respectively with the photon or the $Z$-boson and $g_s$ is the strong coupling constant. Since for the $Z$-boson, the coupling constant, $g_{eL}$, is different for left or right handed projections, $g_{eL}$ depends on $\alpha$.

We see that the diagrams in Fig. 1a and b can be factorized in the sub-diagrams $M_{1a}$, $M_{1b}$ and $M_{2a}$, $M_{2b}$,