Some Comments on Jet Fragmentation Models and $\alpha_s$ Determinations

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Abstract. A number of interrelated topics on jet properties in $e^+e^-$ annihilation is discussed. The need for different $\alpha_s$ values in different fragmentation models is explained, with particular emphasis on the sensitivity to the choice of momentum conservation scheme in independent fragmentation models. Also other factors leading to a broad range of experimental $\alpha_s$ values are discussed. Old and new methods to distinguish different fragmentation models are presented, with particular emphasis on gluon jet fragmentation properties.

1. Introduction

The experimental groups at PETRA and PEP have collected a wealth of data on jets in high energy $e^+e^-$ annihilation events. This way, much has been learned about particle compositions, heavy flavour fragmentation functions, charge correlations, three-jet structures, etc. Despite these advances, the details of jet fragmentation are still fairly poorly known, with no commonly accepted picture emerging so far.

In this paper, we want to discuss some sources of confusion and study possible remedies. First a brief introduction is given to the models so far mostly used to interpret data, the string fragmentation (SF) and independent fragmentation (IF) models (Sect. 2). In particular, the impact of different momentum conservation schemes in IF models is sorted out (Sect. 3). This provides us with an understanding of why different fragmentation models need different $\alpha_s$ values to describe the same data (Sect. 4). As a case example, the energy–energy correlation asymmetry is studied in more detail (Sect. 5). Also uncertainties caused by the use of different matrix element implementations are briefly enumerated (Sect. 6). Finally, the possibility of rejecting some fragmentation models is reviewed, with particular emphasis on the concept of and study of gluon fragmentation (Sect. 7).

Our aim is not to present or interpret experimental data in a quantitative fashion. Rather, we want to point to the qualitative behaviour of data and/or models for some interesting observables. The experimental implications anyhow have to be worked out starting from the actual capabilities of a given detector. To this end, all tools necessary to reproduce the results in this paper are available within the framework of the Lund Monte Carlo (JETSET version 5.2) [1].

2. Fragmentation Models

The Lund model for SF (string fragmentation) has been amply described elsewhere [2, 3], here only a brief summary of the most pertinent features is given. The main idea is to use the massless relativistic string, which provides the simplest causal and Lorentz covariant description of a linear force field, to approximate the linearly confining colour flux tube expected in QCD. The original $q$ and $\bar{q}$ are associated with the endpoints of the string, and gluons are associated with energy and momentum carrying kinks on it. Thus, in a $q\bar{q}g$ event, the string is stretched from the $q$ via the $g$ to the $\bar{q}$. After fragmentation, this will lead to particles lying predominantly along two hyperbolae in momentum space, one in the $qg$ angular range, with the $q$ and $g$ directions as asymptotes, the other correspondingly in the $\bar{q}g$ range. An important and nontrivial feature of the string model is that the fragmentation of a $q\bar{q}g$ event continuously approaches that of a simple $q\bar{q}$ one when the $qg$ or $\bar{q}g$ invariant mass becomes small [3].

The breaking of the string by the production of $q\bar{q}$ pairs is described by the tunnelling mechanism [2]. This leads to the suppression of the production of
heavier flavours, for $s\bar{s}$ by a factor $\sim 1/3$, for $c\bar{c} \sim 10^{-11}$. The same mechanism can be used to give a simple model for baryon production. It also gives a Gaussian $p_L$ spectrum, quantified by the parameter $\sigma^2 = \langle p_L^2 \rangle = 2\sigma^2$. The distribution of breakup vertices, and hence the sharing of energy and momentum between the produced hadrons, is derived from the principle of left-right symmetry [2]. Since all breakup vertices are causally disconnected, the process can be described recursively. In the simple $q\bar{q}$ case one then obtains a scaling function

$$f(z) = \frac{1}{z} \cdot (1 - z)^a e^{-b m_T^2 / z}$$  \hspace{1cm} (1)$$

for the fraction $z$ of remaining $E + p_T$ (i.e. energy and longitudinal momentum) taken by a hadron with transverse mass $m_T$ (in this paper the term fragmentation function is reserved for the actual particle spectrum obtained by recursive use of the scaling function). The extension of (1) to multijet events is discussed in [3]. The two universal parameters $a$ and $b$ are properly to be determined experimentally. We have chosen to keep $a = 1$ fixed in the following; fits then give $b \approx 0.7$ GeV$^{-2}$ [4]. A main consequence of (1) is that heavy hadrons (charm, bottom) obtain harder fragmentation spectra, $\langle z_p \rangle \approx 0.56$ and $\langle z_b \rangle \approx 0.82$, in good agreement with experimental data [5].

The most well known IF (independent fragmentation) model is the one presented by Field and Feynman [6] for single $u$, $d$, and $s$ jets. For applications to $e^+e^-$ annihilation many further components are necessary: charm and bottom fragmentation and decay, gluon jets, QCD matrix elements, etc. Of the many programs written, we restrict our attention to the Hoyer et al. [7] and Ali et al. [8] Monte Carlos, which (in addition to the Lund Monte Carlo) are the only ones to have found extensive experimental use, e.g. for $z_{V}$ determinations. In the Hoyer model, the gluon is assumed to fragment like a quark of random flavour ($u$, $d$, $s$, $\bar{u}$, $\bar{d}$, $\bar{s}$), denoted $g = q$ in the following. Optionally one could use a softer scaling function than for ordinary quark jets (as is actually done in the Hoyer Monte Carlo, although there not much enough to make any real difference), but we will refrain from this here. In the Ali model, the Altarelli–Parisi splitting function [9] is used to divide the gluon energy between a $q$ and a corresponding $\bar{q}$ jet, which then are allowed to fragment independently. This option, $g = q\bar{q}$, thus gives a softer gluon fragmentation function than the $g = q$ one. The (well separated, high energy) Lund gluon actually fragments somewhat softer than the $g = q\bar{q}$ one, since the energy is evenly shared between the two string pieces stretched by the gluon, but basically $g = q$ and $g = q\bar{q}$ may be used to represent “reasonable extremes” for the longitudinal fragmentation properties. The transverse momentum distributions in quark and gluon fragmentation can be chosen independently of each other in IF models (not so in SF, where they are one and the same), but this freedom is used neither here nor in the standard Hoyer and Ali Monte Carlos.

In both the Hoyer and Ali models, the fragmentation is assumed to take place in the hadronic CM frame. This is important because IF is explicitly Lorentz frame dependent, in that a different result would have been obtained had the parton configuration been boosted to another frame, allowed to fragment independently there, and afterwards been boosted back. A more consistent alternative is outlined in [10]: if one assumes separate kinds of quark and gluon “strings”, the relevant frame for a $q\bar{q}$ event is the one where the string tensions exactly balance. Such “IF” models generally tend to give results intermediate to those of the conventional IF ones and the SF one, and will not be considered further here.

3. Momentum Conservation

The concept of IF inevitably leads to the total energy, momentum and flavour not being exactly conserved. For a long time it was thought that this could be corrected for trivially, even to the point that the subject was not even mentioned in model descriptions [7, 8]. We will in the following show why this is not correct.

First consider a massless parton produced with $E = p_L = E_0$. This parton is assumed to fragment independently, so that the massless parton is replaced by a jet with nonzero invariant mass, i.e. $p_L \neq E$. In the basic procedure of [6], the average jet energy is close to the original parton one, $\langle E \rangle \approx E_0$. The mean $p_L$ value is then given by $\langle p_L \rangle = E_0 - E_m$, where $E_m$ the average mismatch between energy and momentum, is independent of $E_0$ for $E_0$ not too small. Approximately, $E_m \approx \langle m \rangle / \langle z \rangle$, where $\langle m \rangle$ is the mean transverse mass of primary hadrons and $\langle z \rangle$ some mean of the scaling variable. This form for $E_m$ can be derived from the assumption of scaling, via the intermediate result that the jet mass-squared grows linearly with the jet energy. A more direct proof is given by the explicit generation of jets, Fig. 1. In particular, a softly fragmenting gluon corresponds to a smaller $\langle z \rangle$ and hence a larger $E_m$.

For a back-to-back two-jet system, the average longitudinal momentum is decreased by the same amount on both sides, such that the total momentum is conserved on the average. Not so for three-jet events. If the total momentum before fragmentation is vanishing, and if the final jet momenta are parallel with the initial parton momenta, then all initial momenta would have to be scaled down by the same factor to keep total momentum conserved. In IF, however, a fixed amount $E_m$ of momentum is subtracted from each jet, such that the relative change is largest for a low-momentum parton, Fig.