Forces acting on induction machine stator core due to winding faults

W. Rams, J. Rusek and J. Skwarczyński, Kraków, Poland

Contents: A newly developed model for direct calculation of the Fourier components of currents is presented. Any connection of stator coils, including shortings, can be analysed. A formula giving forces acting on the stator has the form of a sum of various waves. The waves causing elliptical deformations of the stator yoke have been specially investigated. The calculation and measurement results given refer to a 7.5 kW machine.

Beeinflussung der auf den Stator von Induktionsmaschinen wirkenden Kräfte durch Wicklungsschäden


List of symbols

- \( U, I \) (without subscripts) one-column matrices of voltages or currents (time behaviours)
- \( U, I \) (with two subscripts, e.g. \( U_{23} \) or \( I_{12} \)) complex coefficients of Fourier-series of orders indicated by the subscripts
- \( R, L \) matrices of resistances or inductances
- \( K_x \) column matrix of \( \mu \)-order harmonics of stator-rotor inductances
- \( B \) waves of air-gap flux density
- \( F_{rad,\infty} \) waves of radial forces acting on a unit area of the stator inner surface (in N/m²)
- \( B, P \) (with three indices, e.g. \( B_{x,1,\alpha} \), \( P_{x,\alpha,\beta} \)) components of \( B \) or \( F_{rad,\infty} \) respectively. The space-time orders of these components are indicated by triples of their subscripts.
- \( F_{s,\delta} \) maximal amplitude of the radial second-order force
- \( \mu_0 \) vacuum magnetic permeability
- \( \Theta = \Omega t + \Theta_0 \) angle of rotor revolution
- \( r, \delta, l_{pe} \) air-gap radius, thickness or axial length
- \( y', z' \) span or number of turns of \( i \)-th stator coil
- \( w \) (circumferential) coordinate of the middle of \( i \)-th stator coil
- \( k_{0,i} \) coefficient of stator slot openings for \( \mu \)-order waves

Superscripts
- \( s, r \) stator or rotor
- \( \dot{s} \) stator quantities accounting for constraints imposed by the connection matrix
- \( \dot{r} \) rotor quantities in symmetrical components
- \( i \) number of stator coil or slot
- \( * \) complex conjugate
- \( T \) transpose vector

Subscripts
- \( \mu, \lambda, \gamma \) order numbers referring to the circumferential coordinate \( x \)
- \( \vartheta, \nu, \chi \) order numbers referring to the supply angular frequency
- \( \beta, \gamma, \kappa, \lambda \) order numbers referring to the rotor angular velocity

1 Introduction

Various methods of diagnosis of winding faults are already known [1 - 5], others are still under development. To the latter, applicable to machines with more than one pole-pair, belongs the concept making use of elliptic-type vibrations of the stator core. The concept consists of watching the harmonic spectrum of these vibrations and its changes due to winding damage. To check the usability of the concept mentioned, a procedure of calculating the relevant electromagnetic excitations has been developed. To estimate the level of expected stator yoke deformations the simple free-ring model of the stator has been assumed. Obviously, if the winding comprises short-circuited coils, obtaining reasonable results of calculations requires considering higher harmonics of inductances of the air-gap field [6].

Assuming that the stator yoke can be treated as a free cylindrical body subjected to shear deformations, the magnitudes of elliptical deformations have been calculated [7] and compared with the results of measurements. All the results refer to steady-state operation with constant rotor angular speed \( \Omega \).

2 Voltage equations for steady-state operation

On the stator there are \( N_s \) as yet not connected, coils. In addition, one coil can be divided into two parts, one of which can be short-circuited. In effect, there are \( N_s + 1 \) elementary circuits on the stator.
Assumed symmetry of the cage permits to account for only \( N_r \) rotor elementary circuits, these being the meshes comprising cage bars and ring segments.

Such a system of elementary circuits is described by \((N_s + 1 + N_r)\) differential equations. Making use of a connection matrix \( C \) for the stator and of a transformation into symmetrical components for the rotor, the system equations yield a considerably smaller and simplified system of equations:

\[
\begin{bmatrix}
U^s \\
0
\end{bmatrix} = \begin{bmatrix}
R^s & R' \\
R' & R^s
\end{bmatrix} \begin{bmatrix}
I^s \\
I'
\end{bmatrix} + \frac{d}{dt} \begin{bmatrix}
L^s & L'^s \\
L'^s & L^s
\end{bmatrix} \begin{bmatrix}
I^s \\
I'
\end{bmatrix} + \begin{bmatrix}
W \\
W
\end{bmatrix} \begin{bmatrix}
I^s \\
I'
\end{bmatrix} + \begin{bmatrix}
U_j^s \\
U_j'
\end{bmatrix}
\]

(1)

Matrices \( R^s, R', L^s \) and \( L' \) are constant. Matrices \( L'^s \) and \( L'^s \) depend on rotor angular velocity \( \omega \) and can be expressed:

\[ L'^s = \sum_{\mu = -\infty}^{\infty} K_{\mu} \cdot e^{j\mu\omega t} \cdot [\delta_{\mu,0}N_r, \delta_{\mu,0}N_r + 1, \ldots, \delta_{\mu,0}N_r + (N_r - 1)] \]

(2)

where \( K_{\mu} \) represents one-column vectors. The Kronecker symbols in a row matrix in (2) assume the value of 1 only if an integer \( g \) exists for which both indices of these symbols coincide. Equation (2) indicates that matrix \( L'^s \) is a summation of matrices of \( \mu \)-order inductances each having only one non-zero column \( K_{\mu} \).

For steady-state operation both voltages and currents in (1) can be expanded into complex Fourier series of almost periodic functions [6]:

\[
\begin{bmatrix}
U^s \\
\mathbf{0}
\end{bmatrix} = \sum_{\varrho,k} \begin{bmatrix}
U^s_{\varrho,k} \\
0
\end{bmatrix} e^{j(\varrho\omega + k\Omega)t}
\]

(3)

\[
\begin{bmatrix}
I^s \\
I'
\end{bmatrix} = \sum \begin{bmatrix}
I^s_{\varrho,\ell} \\
I'_{\varrho,\ell}
\end{bmatrix} e^{j(\varrho\omega + \ell\Omega)t}
\]

(4)

where \( \omega \) is the angular velocity of the fundamental harmonic of the supply voltage. In the case of sinusoidal supply discussed here all voltage coefficients in (3) are equal to zero except for \((\varrho, k) = (\pm 1,0)\). Making use of (2), (3) and (4) in (1), the system (5) of algebraic equations presents the harmonic balance model for the machine in steady-state:

\[
\begin{bmatrix}
U^s \\
\mathbf{0}
\end{bmatrix} = \begin{bmatrix}
R^s & R' \\
R' & R^s
\end{bmatrix} \begin{bmatrix}
I^s \\
I'
\end{bmatrix} + \begin{bmatrix}
W \\
W
\end{bmatrix} \begin{bmatrix}
I^s \\
I'
\end{bmatrix} + \begin{bmatrix}
U_j^s \\
U_j'
\end{bmatrix}
\]

(5)

The stator part of (5) contains submatrix equations whereas in the rotor part all the equations listed are single ones. For the sinusoidal supply considered here, system (5) has to be solved for \( \varrho = +1 \) only. Some superscripts in (5) are followed by expression \((\text{mod}\ N_r)\). This means that the values of those superscripts are to be replaced by their modulo-\( N_r \)-values [6], indicating the numbers of the rotor symmetrical components.

The number of equations in (5) is infinite. For practical calculations only a finite number of these equations can be taken into account. Among the equations chosen, there have to appear those referring to the stator current components of order \((\varrho, 0)\) as well as those referring to the rotor current components of order \((\varrho, -p)\). (Here the