Are Dual Models Able to Describe Large-Angle Scattering?

M. Quirós
Instituto de Estructura de la Materia, Serrano, 119, Madrid-6, Spain

Received 13 February 1981; in revised form 17 July 1981

Abstract. The fixed-angle limit of the Mandelstam dual model is proved to behave as \( \frac{d\sigma}{dt} \sim f(\theta)s^{-n} \). The power \( n \) is a function of the masses of external particles and the slope and intercepts of Regge trajectories coupled to direct and crossed channels. The predicted energy dependence is consistent with experimental data at 90° for fifteen two-body reactions with meson trajectories exchanged in the t-channel. However the angular dependence \( f(\theta) \) predicted by the Mandelstam model is not in agreement with the experimental dependence. We thus propose generalized Mandelstam amplitudes, giving a very good description of differential cross-sections for \( pp \) and \( \bar{p}p \) elastic scattering at high energies and large momentum transfers, and with good theoretical properties: crossing, Regge behaviour, duality and polynomial residues. Differential cross-sections for \( pp \) scattering at all angles, the ratio \( \frac{(d\sigma/dt)_{pp}}{(d\sigma/dt)_{\bar{p}p}} \), and effective trajectories and residues (in the limit \( t \to -\infty \)) have been successfully compared with experimental data.

1. Introduction

The inability of the Veneziano model to define a reasonable fixed-angle behaviour, in spite of other phenomenological and theoretical successes, is a trouble to the application of the dual approach. In particular, the four-point Veneziano amplitude \( V_0(s,t) = B(-\alpha(s), -\alpha(t)) \), where \( \alpha(x) = \alpha_0(x) + \alpha'x = 0 \) are Regge trajectory functions and \( B \) the beta function, behaves in the fixed-angle limit, \( \alpha(s) \to \infty, \alpha(t) = b\alpha(s) \), as \([1]\)

\[
V_0(s,t) \propto \exp \{ \alpha(s)(-b \ln (-b) + (1+b) \ln (1+b)) \} \tag{1}
\]

and the fast fall off of the fixed-angle cross-section provided by (1) is in disagreement with current experimental data \([2]\). On the other hand the fixed-angle behaviour of other dual resonance models \([3]\), as the Neveu–Schwarz or the Shapiro–Virasoro model, follow closely the behaviour (1), so that the usual “additive” dual models seem unable to describe the high-energy fixed-angle scattering, and this is a serious objection to these dual models. However, it is important to note that the dual amplitude is certainly not unique, in the sense that the series

\[
A(s,t) = \sum C_n V_n(s,t) \tag{2}
\]

where \( C_n \) are arbitrary coefficients and \( V_n(s,t) = B(-\alpha(s) + n, -\alpha(t) + n) \) Veneziano satellite terms, also satisfies all the FESR, duality and Regge behaviour requirements. But the fixed-angle behaviour of (2) may be qualitatively different from the behaviour (1) of each term, so that the agreement with experimental data is not hopeless.

In Sects. 2 and 3 we shall compute the fixed-angle behaviour of the Mandelstam model \([4]\), which is a particular case of the series (2), and compare its energy dependence with data from a number of reactions whose cross-sections have been measured at 90°.

Let us remind, before going on, that the original motivation of the Mandelstam model was to avoid odd-daughter Regge trajectories in the t-channel. Actually, this seems to be the experimental observation for meson trajectories. In particular, the second daughter of the non-strange degenerate \( p - \omega \) trajectory is well established \([5]\), with a \( p'(1600) \) p-wave resonance and an \( e(1300) \) s-wave resonance strongly coupled to \( \pi \pi \) and lying on it. However, there is not any indication \([6]\) of a \( I^G(J^P) = 1^+(1^-) \), \( p'(1250) \), resonance, coupled to \( \pi \pi \) nor of the existence of the \( \rho^+(0^+) \), \( e(770) \), resonance, so that we must conclude that the first daughter of the \( p - \omega \) trajectory is absent. On the other hand, the second daughter of the strange, \( I = 1/2, K^* \)-trajectory is also well established \([5]\) with the presence of \( \chi(1400) (J^P = 0^+) \) and \( K_0(1700) (1^-) \), while there is not any experimental evidence for the existence of the particles which should lie on the first
with
\[ e' = \frac{\delta}{2} \left( 2 - n + 1 \right) \]

and \( m_i \) are the masses of the external particles. Actually, (4) is a generalization of the Mandelstam determination \([4]\) to the case of unequal external particles and also unequal s and t-channels.

Using (3) in (2) we can give to the Mandelstam amplitude the following integral representation
\[
A(s, t) = \frac{1}{C_n} \int_0^1 dx x^{-\Delta(0) - 1} \left(1 - x \right)^{-\Delta(t) - 1} \left(1 - x(1 - x)\right)^{\delta/2}
\]

2. Fixed-Angle Behaviour of the Mandelstam Model

The high-energy fixed angle behaviour of (5), \( |s| \to \infty \), \( t = bs, b = -\sin^2 \theta/2 \), \( \theta \) being the center of mass scattering angle, can be written as
\[
A(s, \theta) \approx \int_0^1 dx \omega(x)^{-\Delta(s) - 1} \left(1 - x \right)^{a - 1} \left(1 - x \right)^{b/2}
\]

where \( \omega(x) = x(1 - x) \) and \( a = \Delta(t) + b \). Changing variables in (6) from \( x \) to \( z \) as \( x = z/(1 - z) \), we can cast (6) as
\[
A(s, \theta) \approx \int_0^\infty dz \exp \left\{ \frac{1}{2} \left( \frac{1}{2} - a \right) z \right\} g(z)
\]

3. Comparison with Experimental Data

Next, we shall compute the energy dependence predicted by (9) and (4) for a number of two-body reactions with meson trajectories exchanged in the \( s \)-channel and for which experimental data of the differential cross-section at \( \theta = 90^\circ \) are available. Two basic approximations are hidden behind. First, the spin of particles is neglected, following the old good tradition, so that baryons and vector mesons are handled as if they were zero spin mesons. Second, in cases when several trajectories can couple to a given two body channel, only the leading one is taken into account. The masses of the particles will be taken from the Review of Particle Properties \([6]\) and the universal slope of Regge trajectories, \( \alpha' = 0.9 \) (Gev)\(^{-2} \). Since the power \( n \) is a function of the intercepts \( \Delta(t) \) and \( \Delta(t) \), we shall classify the considered reactions according to the meson trajectories exchanged in the \( s \)-channel.

\[ \text{i) Processes with the } \rho \to \omega \text{ Trajectory Coupled to the } t\text{-Channel} \]

The intercept of the natural parity \( I = 0, 1 \) Regge trajectory is taken to be \( \Delta(t) = 0.5 \). The reactions belonging to this group are again classified according to the leading trajectory exchanged in the \( s \)-channel.