Are Dual Models Able to Describe Large-Angle Scattering?

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Abstract. The fixed-angle limit of the Mandelstam dual model is proved to behave as \( \frac{d\sigma}{dt} \sim f(\theta)s^{-n} \). The power \( n \) is a function of the masses of external particles and the slope and intercepts of Regge trajectories coupled to direct and crossed channels. The predicted energy dependence is consistent with experimental data at 90° for fifteen two-body reactions with meson trajectories exchanged in the t-channel. However the angular dependence \( f(\theta) \) predicted by the Mandelstam model is not in agreement with the experimental dependence. We thus propose generalized Mandelstam amplitudes, giving a very good description of differential cross-sections for \( pp \) and \( \bar{p}p \) elastic scattering at high energies and large momentum transfers, and with good theoretical properties: crossing, Regge behaviour, duality and polynomial residues. Differential cross-sections for \( pp \) scattering at all angles, the ratio \( \frac{d\sigma}{dt}_{pp}/d\sigma/dt_{\bar{p}p} \) and effective trajectories and residues (in the limit \( t \to -\infty \)) have been successfully compared with experimental data.

1. Introduction

The inability of the Veneziano model to define a reasonable fixed-angle behaviour, in spite of other phenomenological and theoretical successes, is a trouble to the application of the dual approach. In particular, the four-point Veneziano amplitude \( V_0(s,t) = B(-\alpha(s), -\alpha(t)) \), where \( \alpha(x) = \alpha_x(0) + \alpha_x'x \) are Regge trajectory functions and \( B \) the beta function, behaves in the fixed-angle limit, \( \alpha(s) \to \infty, \alpha(t) = b\alpha(s) \), as [1]

\[
V_0(s,t) \cong \exp \{ \alpha(s)(-b \ln(-b) + (1+b) \ln(1+b)) \}
\]

and the fast fall off of the fixed-angle cross-section provided by (1) is in disagreement with current experimental data [2]. On the other hand the fixed-angle behaviour of other dual resonance models [3], as the Neveu-Schwarz or the Shapiro-Virasoro model, follow closely the behaviour (1), so that the usual "additive" dual models seem unable to describe the high-energy fixed-angle scattering, and this is a serious objection to these dual models. However, it is important to note that the dual amplitude is certainly not unique, in the sense that the series

\[
A(s,t) = \sum C_n V_n(s,t)
\]

where \( C_n \) are arbitrary coefficients and \( V_n(s,t) = B(-\alpha(s) + n, -\alpha(t) + n) \) Veneziano satellite terms, also satisfies all the FESR, duality and Regge behaviour requirements. But the fixed-angle behaviour of (2) may be qualitatively different from the behaviour (1) of each term, so that the agreement with experimental data is not hopeless.

In Sects. 2 and 3 we shall compute the fixed-angle behaviour of the Mandelstam model [4], which is a particular case of the series (2), and compare its energy dependence with data from a number of reactions whose cross-sections have been measured at 90°.

Let us remind, before going on, that the original motivation of the Mandelstam model was to avoid odd-daughter Regge trajectories in the t-channel. Actually, this seems to be the experimental observation for meson trajectories. In particular, the second daughter of the non-strange degenerate \( \rho - \omega \) trajectory is well established [5], with a \( \rho'(1600) \) p-wave resonance and an \( \epsilon(1300) \) s-wave resonance strongly coupled to \( \pi\pi \) and lying on it. However, there is not any indication [6] of a \( I^G(J^P) = 1^+(1^-) \), \( \rho'(1250) \), resonance, coupled to \( \pi\pi \) nor of the existence of the \( 0^+(0^+) \), \( \epsilon(770) \), resonance, so that we must conclude that the first daughter of the \( \rho - \omega \) trajectory is absent. On the other hand, the second daughter of the strange, \( I = 1/2, K^* \)-trajectory is also well established [5] with the presence of \( \chi(1400) \) (\( J^P = 0^+ \)) and \( K_s(1700) \) (\( 1^- \)), while there is not any experimental evidence for the existence of the particles which should lie on the first
The requirement of absence of odd-daughter Regge trajectories exchanged in the \( t \)-channel leads to the following value for the coefficients

\[
C_n = \frac{(-1)^n \delta}{n!} \left( \frac{\delta}{2} - 1 \right) \ldots \left( \frac{\delta}{2} - n + 1 \right)
\]

(3)

with

\[
\delta = \alpha' \sum_{i=1}^{4} m_i^2 + a_i(0) + 2a_i(0) + 1
\]

(4)

and \( m_i \) are the masses of the external particles. Actually, (4) is a generalization of the Mandelstam determination [4] to the case of unequal external particles and also unequal \( s \) and \( t \)-channels.

Using (3) in (2) we can give to the Mandelstam amplitude the following integral representation

\[
A(s, t) = \int_0^1 dx x^{-\alpha(s) - a - 1} \left( 1 - x(1 - x) \right)^{b/2}
\]

(5)

2. Fixed-Angle Behaviour of the Mandelstam Model

The high-energy fixed angle behaviour of (5), \( |s| \to \infty \), \( t = bs \), \( b = -\sin^2 \theta / 2 \), \( \theta \) being the center of mass scattering angle, can be written as

\[
A(s, \theta) \sim \int_0^1 dx \omega(x)^{-\alpha(s) - 1} \left( 1 - x(1 - x) \right)^{b/2}
\]

(6)

with \( \omega(x) = x(1 - x)^b + a = a_0(0) - bx_0(0) \). Changing variables in (6) from \( x \) to \( z \) as \( \exp(-z) = \omega(x) \), i.e. \( x = x(z) = \omega^{-1}(e^{-z}) \), we can cast (6) as

\[
A(s, \theta) \sim \int_0^\infty dz \exp\left( \alpha(s) + 1 \right) g(z)
\]

(7)

where

\[
g(z) = \left( 1 - x \right)^{b-a} \left( 1 - b + 1 \right)^{-1} \left( 1 - x(1 - x) \right)^{b/2}
\]

Let us suppose for the moment that \( \text{Im} s > 0 \). It means that we shall compute the asymptotic limit along a complex direction in the \( s \)-plane. Then we can close the real axis \( (-\infty, +\infty) \) by a semicircle of radius \( r \to \infty \), having its centre at the origin, above the real axis. The new contour can be deformed to the sum of paths encircling the singularities of the integrand of (7). The asymptotic behaviour of (7) is, thus, governed by the singularities of the function \( g(z) \). These singularities are of two kinds [3, 7]: i) The function \( x(z) \) has cuts in the complex \( z \)-plane, with branch points at \( \text{Re} z = -\ln b(1 + b)^{-1-b} \), \( \text{Im} z = \pm \pi b + 2n \pi \) (\( n = 0, \pm 1, \ldots \)). These singularities give rise to an exponentially decreasing behaviour for the amplitude, similar to (1), and will not be considered in detail, and ii) The function \( \left( 1 - x(1 - x) \right)^{b/2} \), when \( \delta \) is not an even number, has two cuts with branch points at \( x = e^z \), \( z \in \mathbb{C} \). The presence of these cuts is the essential feature making the asymptotic behaviour of the series (5) qualitatively different and letting the large-angle dual amplitude to be compared with the experimental data. After the change of variables, the corresponding function of \( z \) has cuts with branch points of \( \text{Re} z = 0 \), \( \text{Im} z = \pm \pi n \pm \pi (b - 1)/3 \), \( n = 0, \pm 1, \ldots \). Taking the limit \( |s| \to \infty \) along the complex direction \( \text{Im} s = \varepsilon \text{Re} s \), the contribution of these singularities to the asymptotic behaviour is given by [7]

\[
A(s, \theta) \approx \Gamma^{-1} \left( -\delta/2 \right) \left( 1 + b + b^2 \right)^{-(1 + \delta/2)/2} \left| s \right|^{-1 - \delta/2} e^{-\pi(1 - b)\varepsilon s/3}
\]

(8)

which is dependent on the direction \( \varepsilon \) of the ray. However, the limit \( \varepsilon \to 0 \) in the above expression is smooth, so that we can approach the physical (real) axis putting \( \varepsilon = 0 \). In this way the behaviour predicted by the Mandelstam model for the differential cross-section at large angle has the following factorized from

\[
d\sigma/dt = f(\theta)s^{-n}
\]

(9)

where \( f(\theta) = \Gamma^{-2} \left( -\delta/2 \right) (3 + \cos^2 \theta)^{-1 - \delta/2} \) and \( n = \delta + 4 \). This factorization is the typical behaviour predicted by parton models, as quark counting rules [8], the essential difference being that the power \( n \) predicted by constituent models is an integer number determined by the total number of valence quarks, or active fields, participating in the reaction, while the exponent predicted by the Mandelstam model is a function of the masses of the external particles and the slope and intercepts of Regge trajectories exchanged in \( s \) and \( t \)-channels.

3. Comparison with Experimental Data

Next, we shall compute the energy dependence predicted by (9) and (4) for a number of two-body reactions with meson trajectories exchanged in the \( t \)-channel and for which experimental data of the differential cross-section at \( \theta = 90^\circ \) are available. Two basic approximations are hidden behind. First, the spin of particles is neglected, following the old good tradition, so that baryons and vector mesons are handled as if they were zero spin mesons. Second, in cases when several trajectories can couple to a given two body channel, only the leading one is taken into account. The masses of the particles will be taken from the Review of Particle Properties [6] and the universal slope of Regge trajectories, \( \alpha' = 0.9 (\text{GeV})^{-2} \). Since the power \( n \) is a function of the intercepts \( \alpha(0) \) and \( \alpha(0) \), we shall classify the considered reactions according to the meson trajectories exchanged in the \( t \)-channel.

i) Processes with the \( \rho - \omega \) Trajectory Coupled to the \( t \)-Channel

The intercept of the natural parity \( I = 0, 1 \) Regge trajectory is taken to be \( \alpha(0) = 0.5 \). The reactions belonging to this group are again classified according to the leading trajectory exchanged in the \( s \)-channel.