Proton Decay in the $SU(4)_c \times SU(2)_L \times SU(2)_R$ Model

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Abstract. Possible proton decays in the $SU(4)_c \times SU(2)_L \times SU(2)_R$ unification model are discussed. There are some characteristics in the decay products, which are different from those in the standard $SU(5)$ or $SO(10)$ model, in certain cases.

Some twenty years ago when people found that the strong interactions respected approximately the $SU(3)$ symmetry, there were some attempts to enlarge the symmetry group. The symmetry of the then strong interactions was first enlarged to the $SU(6)$, which included an $SU(2)$ spin as its subgroup, then to the $SU(12)$, which encompassed the Lorentz group as its subgroup. It turned out that the $SU(12)$ was a bad symmetry and the $SU(6)$ was not as good as the $SU(3)$ with respect to experimental data. In the quark model, the above $SU(3)$ symmetry turns out to be an accidental global symmetry of QCD-Lagrangian with three light fermions. Now, when we study unification gauge theories, we may be concerned with how big the "real size" of the unification gauge group is. The largest group might not be the best which fits either simple group. In this note we will study proton decays in the gauge model

$$SU(4)_c \times SU(2)_L \times SU(2)_R$$

(1)

where $SU(4)_c$ is the Pati-Salam symmetry [1]. As some composite models of leptons and quarks naturally give the symmetry (1) [2], this unification model becomes more interesting*

We suppose, following Pati-Salam, that the first step of symmetry breaking at about the scale larger than $10^5$ GeV is

$$SU(4)_c \rightarrow SU(3)_c \times U(1)_{B-L}$$

(2)

* Some of these composite models have $SU(3) \times SU(2) \times SU(2) \times U(1)$ symmetry

Therefore our results can be easily transfered of fit the following unification model

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

(3)

Proton decays in $SU(3)_c \times SU(2) \times U(1)$ gauge theories as the low energy effective theory of a grand unification theory has been discussed by many authors [3]. Implicitly they suppose that the grand unification group is larger than or equal to the $SU(5)$. The difference between our work and theirs is that we see the group (1) as the original symmetry but not a survived symmetry from a breaking of a larger symmetry. Thus we do not have gauge bosons (light or heavy) other than those in the adjoint representation of the group (1). As is well known, if quarks are fractionally charged, none of these vector bosons contribute to proton decays and Higgs mediated proton decays (if they exist) become the main decay modes. Since Higgs-Fermion couplings (the Yukawa couplings) are much weaker than the gauge coupling, proton decays are suppressed. However, there are no principles as the $\partial \mu \rightarrow D_\mu$ rule in the gauge coupling case to govern the strengths of the Yukawa couplings, and we cannot estimate the decay rate as in the case of gauge boson mediated processes. Our purpose in this note is to find some proper characteristics of these proton decays which may help to say something about the unification group once proton decays [4] are observed. We find our aim is partially reached.

Under the symmetry group (1), fermions $\psi_L$ and $\psi_R$ are in the representations $(4, 2, 1)$ and $(4, 1, 2)$ respectively and we suppose that there are three generations of fermions. The Higgs in the adjoint representation of $SU(4)$ does not contribute to the proton decay. Therefore we need only to consider the Higgs in the sextet or decuplet. These Higgs bosons could be effective Higgs bosons, i.e. a few Higgs multiplets whose products are in $6$ or $10$. Let us first
discuss the Higgs multiplets in sextets of the \( SU(4) \).

The relevant Lagrangian is

\[
\mathcal{L} = g_{\xi\eta} \varepsilon^{\mu\nu} \bar{\phi}_{[ij]} \phi_{[ij]} \psi_{\text{Liaa}} \psi_{\text{Lja}} \eta_{\text{Ljbln}} \\
+ G_{\xi\eta} \varepsilon^{\mu\nu} \phi_{[ij]} \phi_{[ij]} \psi_{\text{Liaa}} \psi_{\text{Lja}} \eta_{\text{Ljbln}} \\
+ G'_{\xi\eta} \varepsilon^{\mu\nu} \phi_{[ij]} \phi_{[ij]} \psi_{\text{Lja}} \psi_{\text{Liaa}} \eta_{\text{Ljbln}} \\
+ G_{\xi\eta} \varepsilon^{\mu\nu} \phi_{[ij]} \phi_{[ij]} \psi_{\text{Liaa}} \psi_{\text{Lja}} \eta_{\text{Ljbln}} \\
+ (L \to R) + (H \cdot C)
\]

(4)

where \( i, j, k, l = 1, 2, 3, 4 \) are the \( SU(4)_c \) color indices, \( a, b = 1, 2 \) are the \( SU(2)_L \) indices, \( \xi, \eta = 1, 2 \) are the Lorentz indices and \( \xi, \eta = e, \mu, \tau \) are generation indices. To meet the Fermi statistics we have for the coupling constants

\[
g_{\xi\eta} = -g_{\eta\xi}, \quad G_{\xi\eta} = -G_{\eta\xi} \quad (5)
\]

The Lagrangian (4) is left-right symmetric.

After the spontaneous symmetry breaking, we will get: 1) Mass splitting between \( \phi_{[ij]} \) and \( \phi_{[ij]} \), probably the former is much lighter than the latter; 2) Mixings between components \([m,n]\) and charge conjugated components \([m,n+1]\), where \( m, n, p = 1, 2, 3 \) cyclic are the \( SU(3)_c \) indices. Since the sextet is a real representation of \( SU(4) \), this mixing could be as big as any. Obviously, those vertices with \( g_{\xi\eta} \) and \( G_{\xi\eta} \) are left-right asymmetric after symmetry breaking. We can prove that terms with \( g_{\xi\eta} \) and \( G_{\xi\eta} \) still remain symmetric for charged fermions. For example,

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g'_{\xi\eta} = g_{\xi\eta}, \quad G'_{\xi\eta} = G_{\xi\eta}
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\]

which is superficially left-right symmetric with unpolarized fermions, where

\[
\psi = \psi_L + \psi_R, \quad \psi^C = i\gamma_2 \gamma_4 \psi^T
\]

Now this characteristic of (6) will not get changed when fermions are getting massive. The reason is they get their masses exclusively from Higgs \( \phi_{[ab]} \), which is in \( (1, 2, 2) \) or \( (15, 2, 2) \) of the group (1). The Yukawa coupling which generates the masses of charged fermions reads

\[
\bar{\psi}_{\text{Liaa}}(a_{\xi\eta} \phi_{[ab]} + b_{\xi\eta} \bar{\phi}_{[ab]}) \psi_{\text{Ria}} \\
+ \bar{\psi}_{\text{Ria}}(a_{\xi\eta} \phi_{[ab]} + b_{\xi\eta} \bar{\phi}_{[ab]}) \psi_{\text{Liaa}} + (L \leftrightarrow R)
\]

(7)

where \( \bar{\phi} = \tau_2 \phi^* \tau_2 \), and \( a_{\xi\eta}, b_{\xi\eta} \) are coupling constants. Let

\[
a_{\xi\eta}' = a_{\xi\eta} + a_{\xi\eta}^*, \quad b_{\xi\eta}' = b_{\xi\eta} + b_{\xi\eta}^*
\]

and let \( \langle \phi \rangle = \text{diag}[k, k'] \) with real \( k \) and \( k' \) (which can be realized by a suitable \( SU(2)_L \times SU(2)_R \) rotation), we find that the mass matrices of the fermions are as follows

\[
M_{\xi\eta} = a_{\xi\eta}' k + b_{\xi\eta}' k' \quad (T_3 = \frac{1}{2} \text{ fermions})
\]

\[
M_{\xi\eta} = a_{\xi\eta}' k' + b_{\xi\eta}' k \quad (T_3 = -\frac{1}{2} \text{ fermions})
\]

(9)

and are both Hermitian; \( \psi_L \) and \( \psi_R \) will subject to the same transformation when going into mass eigenstates. Therefore (6) will remain the same as in the left-right symmetric case in terms of mass eigenstates. However this is usually not the case for neutrinos. As we know, \( (10, 3, 1) \) and \( (10, 1, 3) \) may contribute Majorana masses to left and right neutrinos respectively in an asymmetric way.

We will assume that

\[
|g_{\xi\eta}|^2 >> |g_{\eta\xi}|^2, \quad |G'_{\xi\eta}|^2 >> |G_{\eta\xi}|^2 \quad (\xi + \eta)
\]

(10)

Thus if the dominant proton decay is mediated by \( \phi_{[ij]} \) then we will have

\[
p \rightarrow e^+ \pi^0, \quad (e_L)^C K^+ \quad \text{or} \quad (e_R)^C K^+
\]

(11)

where \( e^+ \) is not polarized. If the \( \phi_{[ij]} \) and \( \phi_{[ij]} \) mixing contributes mainly, we will have

\[
p \rightarrow e^+ K^0, \quad (e_L)^C K^+ \quad \text{or} \quad (e_R)^C K^+
\]

(12)

Notice that these processes are semi-nondiagonal, i.e. one of the two decay products is in the second generation. If \( \phi_{[ij]} \) dominates the proton decay, we will have

\[
p \rightarrow (\mu_L)^C K^0, \quad (\mu_L)^C K^+
\]

(13)

These processes are nondiagonal. Each of the proton decay processes listed above has distinguishable decay products: either with an unpolarized electron or with dominantly a muon and/or kaon. Incidentally, the proton decay in the standard models like the \( SU(5) \) [5] or \( SO(10) \) [6] is expected to produce a polarized electron* and to be dominantly diagonal processes.

As we have indicated in the beginning, it is not in our range of ability to estimate the decay rate of the processes. However, in order to give some concrete concept on the parametrization in the Lagrangian (4), let us discuss two typical diagrams. One of them is given in Fig. 1 whose rate is

\[
\Gamma \sim \frac{1}{M_p^2} g_{ee}^2 G_{ee}^2 m_p^2
\]

(14)

* In the \( SU(5) \) model \( \langle e_L - e_R \rangle = \frac{1}{2} \) \( \langle e_L + e_R \rangle = \frac{3}{2} \)