Temporal Gauges and Time Boundary Conditions

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Abstract. A new gauge of temporal nature characterized by $A_0=0$ is discussed. This gauge is particularly useful in dealing with static problems with periodic or zero boundary conditions and in treating finite temperature field theory. An application is given to the computation of the Wilson loop to fourth order and the relation to other approaches in analyzed.

1. Introduction

Much attention has been recently devoted to the axial gauges, which in dealing with several problems allow a more direct physical insight [1, 2].

If one deals with periodic boundary conditions, as they arise in finite temperature field theory or as they are frequently used in lattice calculations, one sees that gauges of the type $n_a A^a=0$, $n^2=0$ cannot be directly used; in fact e.g. for $A_0=0$ (temporal gauge) a gauge invariant quantity like the Wilson loop, i.e. a rectangular loop with vertices $(0,-\frac{1}{2\epsilon}),\left(0,\frac{1}{2\epsilon}\right),\left(L,\frac{1}{2\epsilon}\right),\left(L,-\frac{1}{2\epsilon}\right)$, where $\frac{1}{\epsilon}$ denotes the size of the temporal box, reduces to

$$\text{Pexp}i g \int A_0 dx_0$$

which owing to the periodic boundary conditions in time, is identically equal to 1. (The same remark holds obviously for $A_3=0$, with periodic boundary conditions in space.)

A solution to this problem is to avoid periodic boundary conditions and to select the initial and final field configurations in order to satisfy Gauss's law [2, 3]. In such an approach, which was particularly developed by Rossi and Testa [3], one is forced to a formalism non invariant under translations and to rather complex rules for computing the perturbation expansion; in particular new vertices have to be computed to each order perturbation theory, to satisfy the boundary conditions on the fields. A similar disadvantage was found in [4], where in order to obtain the correct exponentiation of the potential in the calculation of the Wilson loop, one had to discard the principal value propagator and adopt a non translationally invariant formalism.

In this paper we discuss a new gauge [5] which allows a translationally invariant formalism (in addition to the possibility of dealing with other time non translationally invariant boundary conditions) where $P \text{exp} i g \int A_0 dx_0$ is not constrained to be identically equal to 1, thus allowing the treatment, among others, of finite temperature field theory [6]. $A_0$ however will maintain a special status which renders this gauge particularly suitable in dealing with static problems or for more general questions in which the time direction plays a preferred role.

$A_0=0$ is the condition on $A_0$ which is nearest to the vanishing of the $A_0$ field, without having the contradictory situation in which all thermal (Polyakov) loops [7] in the time direction

$$\text{Tr} P \text{exp} i g \int A_0 dx_0$$

are identically 1 [8]. Particularly useful will be the fact that for $A_0=0$, $P \text{exp} i g \int A_0 dx_0$ reduces to $\text{exp} i g T A_0(x)$.

The $A_0=0$ gauge admits a residual gauge invariance under the time independent gauge transformations; as usual such a local gauge invariance
makes the propagator ill defined. A well defined propagator is obtained either by introducing a gauge fixing term or by breaking such a residual gauge invariance by imposing proper boundary conditions, adding in both cases the relative Faddeev-Popov ghost term. Both alternatives will be considered here. At the classical level, with \( A_0 = 0 \) we are in presence of a canonical theory where \( A_4 \) play the role of the coordinates and \( A_0 \) that of constant parameters (constant background field).

In this paper in order to have a test on the consistency of the approach, we shall apply this gauge to the computation of the Wilson loop and the thermal Polyakov loops. The formalism is well suited for dealing with finite times, which avoids the ambiguity about the origin of infrared divergent terms. The results for the static potential which we calculate to order \( g^4 \), agree, finite terms included, with those one would obtain in the Feynman and in the Coulomb gauges.

We shall defer to a subsequent paper the application of this gauge to finite temperature field theory. Here we shall bound ourselves to the development of the calculational techniques in this new gauge and to the relation of this \( A_0 = 0 \) gauge to other "temporal gauge" approaches.

The paper is organized as follows: In Sect. 2 we introduce the Lagrangian, discuss Gauss's law, gauge conditions and give the Feynman rules. In Sect. 3 we give the calculation of the Wilson's loop and Polyakov's loops. In Sect. 4 we give a detailed discussion of the results and of their dependence on the size of the time box. Finally in Sect. 5 we compare the present approach to other "temporal gauge" formulations.

2. Lagrangian and Feynman Rules

We shall work directly in euclidean space of 1+3 \(-2\sigma\) dimensions, in view of applications to finite temperature field theory. We start from the usual Yang-Mills lagrangian where \( \frac{\partial A_4}{\partial x_4} = 0 \). The action is

\[
S = \frac{1}{2e} \int dx_4 d\mathbf{x} [-\frac{1}{2} \partial_\alpha A_\alpha \partial_\beta A_\beta - \frac{1}{2} G_{ij}^\alpha G_{ij}^\alpha - \frac{1}{2} f^{abc} A_\alpha A_\beta A_\gamma]
\]

where

\[
G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - g f^{abc} A_\mu A_\beta A_\gamma
\]

\( \mu = 1, 2, 3, 4 \). In this paper we shall work with periodic or zero boundary conditions.

\( S \) is invariant under all gauge transformations which take over from \( A_4 = 0 \) to \( A_4 = 0 \); in particular under all time independent gauge transformations.

As far as Gauss's law is concerned, we can show that it can be derived from such a lagrangian. In fact the equations of motion are

\[
\begin{align*}
D_4 G_{ja} &= 0 \\
D_4 G_{aj} + D_4 G_{ij} &= 0.
\end{align*}
\]

From (2.3b) we derive

\[
D_4 D_4 G_{aj} = i D_4 D_j E_j = 0
\]

which due to the periodic boundary conditions, gives

\[
D_j E_j(x, x_4) = c(x).
\]

Taking the time average and comparing with (2.3a) we get Gauss's law

\[
D_j E_j = 0.
\]

\( S \) can be rewritten as

\[
S = \int \frac{1}{2} e \int dx_4 d\mathbf{x} [-\frac{1}{2} \partial_\alpha A_\alpha \partial_\beta A_\beta - \frac{1}{2} G_{ij}^\alpha G_{ij}^\alpha - \frac{1}{2} f^{abc} A_\alpha A_\beta A_\gamma + \frac{1}{2} G_{ij}^\alpha G_{ij}^\alpha + \frac{1}{2} f^{abc} A_\alpha A_\beta A_\gamma - \frac{1}{2} f^{abc} A_\alpha A_\beta A_\gamma]
\]

Due to the residual gauge invariance the propagators derived from (2.8) would be ill defined. In order to break the residual gauge invariance the simplest procedure would be to introduce a boundary condition of the type \( A_4 \left( x, \pm \frac{1}{2\sigma} \right) = 0 \).

This would possess the advantage of not producing Faddeev-Popov ghosts. However for perturbative calculations it is more practical to introduce gauge conditions on the longitudinal part of the propagator. For example we can use a fixing on the zero mode of the longitudinal field by adding to the action a term of the type [7]

\[
S_B = \frac{e}{2a} \int dx C^a(x) \frac{1}{\nu^2} C^a(x)
\]

with

\[
C^a(x) = \partial_\alpha \tilde{A}_\alpha^a(x)
\]

where