Coupling Constants and Transition Potentials for Hadronic Decay Modes of a Meson

E. van Beveren
Institute for Theoretical Physics, University of Nijmegen, NL-6525 ED Nijmegen, The Netherlands

Received 1 August 1983

Abstract. Within the independent harmonic oscillator model for quarks inside a hadron, a rigorous method is presented for the calculation of coupling constants and transition potentials for hadronic decay, as needed in a multi-channel description of mesons.

1. Introduction

In several publications [1-4] it is discussed that hadronic decay cannot be ignored in hadron models. As a consequence it is essential for the description of mesons (and baryons) to know the allowed decay modes and their relative strengths. The use of independent harmonic oscillators for the calculation of the angular momentum part of rearrangement matrix elements has been demonstrated in a previous paper [5]. There it was shown that the difficulties, which arise in naive recoupling schemes [6] if for a decay process the decay products have internal angular excitations, can easily be handled once one introduces explicit wave functions for the partons involved. This fact has already been elaborated in [7] and [8] and successfully applied to the calculation of branching ratios for the decay of mesons. Here we want to use this as a method for determining the coupling constants and transition potentials in a multichannel meson-meson scattering model [9, 10]. There is, however, in principle no limit on the possible decay modes in the schemes of [7] and [8]. For this reason we prefer the rearrangement method of [5]. Essentially if one wants to calculate all possible decay channels of a given initial state, the harmonic oscillator treatment has the advantage that their number is finite and in a certain sense complete, so that one can always easily check whether one has taken into account all possibilities.

In this paper we will calculate explicitly the matrix elements for the decay processes: meson→meson+meson (Sects. 2-4). Of course, the Clebsch-Gordannery is straightforward, but it is useful to have the complete formulas written somewhere. Moreover, the here presented strategy is not totally trivial. We will see that in order to obtain the proper selection rules for the decay products, we only need to impose Fermi statistics on the quarks and the antiquarks. G-parity is then automatically obeyed.

In Sect. 5 some of the resulting coupling constants are compared to the data.

The construction of a transition potential which can be used in models [9, 10] for the description of hadronic decay, is given in Sect. 6.

2. The Meson just Before Decay

Just before decay a meson is assumed to consist of four partons in the $^3P_0$-model (Fig. 1):

![Fig. 1. Meson just before decay](image-url)
(i) the original quark-antiquark \((q\bar{q})\) pair which carries all the quantumnumbers of the decaying meson,

(ii) the newly created \(q\bar{q}\)-pair with the quantumnumbers of the vacuum \((J^{PC}=0^+).\)

First we introduce a quarkspinor which describes a quark with flavorindex \(a\) (here the number of flavors is restricted to three; the generalization to more flavors is straightforward), colorindex \(x\) and spin magnetic quantumnumber \(\mu\) by

\[
q_a^x(\mu). \tag{2.1}
\]

For an antiquark we use similarly the notation

\[
\bar{q}_a^x(\mu). \tag{2.2}
\]

The wave function describing the system of four partons composing a decaying meson is the product of two parts:

(i) The wave function of the original \(q\bar{q}\)-pair (a color singlet \(q\bar{q}\)-pair with (flavor, spin) indices \((a, m_a)\) for the quark and \((b, m_b)\) for the antiquark; the antiquark is at relative position \(r_{12}\) with respect to the quark; this system has relative angular momentum \(l\), total spin \(s\), total angular momentum \(J\) (z-component: \(J_z\)) and radial excitation of the spatial relative motion \(n\),

\[
\chi_{\text{meson}}(J, J_z; l, s, n; a, b; r_{12}) = \sum_{\mu_1, \mu_2} C_{\mu_2, \mu_1}^a s J C_{\mu_2, \mu_1}^b s J \phi_{n, l, \mu}(r_{12}) \cdot \frac{1}{\sqrt{3}} q_a^x(\mu_1) \bar{q}_b^x(\mu_2). \tag{2.3}
\]

Here and in the following we adopt the standard convention for repeated indices.

(ii) Similarly the newly created color and flavor singlet \(3P_0\)-pair is described by

\[
\chi_{3P_0}(r_{34}) = \sum_{\mu_3, \mu_4} C_{\mu_2, \mu_3}^{1,1} C_{\mu_4, \mu_2}^{1,1} \phi_{0, l, \mu}(r_{34}) \cdot \frac{1}{\sqrt{3}} q_3^x(\mu_3) \bar{q}_4^x(\mu_4), \tag{2.4}
\]

where we have taken the lowest radial excitation in the spatial part. The precise definition of the harmonic oscillator wave functions \(\phi_{n, l, m}(r)\) can be found in [5].

The total wave function for the four particles must be anti-symmetric with respect to the interchange of either two quarks or two antiquarks. For this purpose we define the exchange operator \(P_{ij}\) which operator interchanges partons \(i\) and \(j\).

This way we obtain for the wave function for a meson just before decay

\[
(1 - P_{12} - P_{24} + P_{13} P_{24}) |M \rangle + 3 P_0 |J, J_z; l, s, n; a, b; r_{12}, r_{34}, r_{12, 34} \rangle, \tag{2.5}
\]

where

\[
|M + 3 P_0; J, J_z; l, s, n; a, b; r_{12}, r_{34}, r_{12, 34} \rangle = \chi_{\text{meson}}(J, J_z; l, s, n; a, b; r_{12}) \cdot \chi_{3P_0}(r_{34}) \phi_{0, l, n}(r_{12, 34}). \tag{2.6}
\]

In (2.6) the relative motion of the two \(q\bar{q}\) pairs is assumed to have the ground state quantum numbers. So far we have not taken into account normalization factors, but we will come back to this point in Sect. 4.

3. The Meson-Meson Final State

If out of the two \(q\bar{q}\)-pairs two new mesons are formed, we have a system of two distinct objects each of which consisting of a color singlet \(q\bar{q}\)-pair (Fig. 2). We will not treat here possible octet \(q\bar{q}\)-pairs.

The quantum numbers (total angular momentum, orbital angular momentum, spin and radial orbital excitation) of the mesons are \((J_a, l_a, s_a, n_a)\) for meson 1 and \((J_b, l_b, s_b, n_b)\) for meson 2 and their flavor indices are \((a', d')\) and \((c', b')\) respectively.

The wave function must be symmetric under the interchange of the two mesons, or in terms of the quarks, symmetric under the simultaneous exchange of quarks and antiquarks:

\[
(1 + P_{13} P_{24}) |M + M; J, J_z; l, s, n; a, b; l_a, l_b, s_a, s_b, n_a; J_b, l_b, s_b, n_b; a', d', c', b' ; r_{14}, r_{32}, r_{14, 32} \rangle, \tag{3.1}
\]

where

\[\text{Fig. 2. Two meson final state}\]