Low Mass-Scale Parity Restoration in Expanded Gauge Theories

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Abstract. It is shown that schemes of grand unification with $SU(2n)^4$ gauge symmetry permit the embedding of the left-right symmetric $SU(2)_L \times SU(2)_R \times U(1) \times SU(3)$ intermediate symmetry at relatively low energies [between 250 GeV and a few TeV] as well as allows light mass-scale masses ($\leq 10^5$ TeV) if $n \geq 3$ for values of the weak angle $\sin^2 \theta_w$ and the strong coupling $\alpha_s$ in the ranges $0.20 \leq \sin^2 \theta_w \leq 0.25$, $0.10 \leq \alpha_s \leq 0.15$.

The principal motivation for considering the $SU(2)_L \times SU(2)_R \times U(1)$ gauge symmetry [1] for electroweak interactions is the belief that the observed non-conservation of parity in weak interactions is spontaneous rather than intrinsic in character. In this framework the theory conserves parity once energies of order or beyond the mass-scale $M_w$ that characterises the masses of the right-handed charged gauge bosons ($W^+_R$) are reached. Presently available data on various nuclear $\beta$-decay processes provides a lower bound of 250 GeV [2] on the mass $M_{W_R}$ of $W^+_R$ and the hope is that $M_{W_R}$ is in fact only a few times heavier than $M_{W_L}$, the mass-scale of ordinary weak interactions [$M_{W_L} = (\alpha_m/G_F)^{1/2} \sin^2 \theta_w$]. However, this simple and highly appealing picture in which $M_{W_R}$ is as low as 250 GeV is severely distorted [3] if one invokes the hypothesis that the $SU(2)_L \times SU(2)_R \times U(1) \times SU(3)^f$ electroweak and strong interactions are part of a 'simple' fundamental unifying symmetry $G$ with one coupling constant. The simple descent of $G$

$$G \rightarrow M_w \times SU(2)_L \times SU(2)_R \times U(1) \times SU(3)$$

leads to phenomenologically unacceptable predictions for the unifying mass-scale $M_w$ and the weak angle $\sin^2 \theta_w$;

$$\ln(M_w/M_{W_L}) = \frac{4\pi}{11} \left( \frac{1}{\alpha_m^2(M_{W_L})} - \frac{1}{\alpha_s^{-1}(M_{W_L})} \right),$$

$$\sin^2 \theta_w = 1/4 + \alpha_m(M_{W_L})/\alpha_s(M_{W_L}).$$

For $\alpha_m(M_{W_L}) = e^2/4\pi \approx (128.5)^{-1}$, $\alpha_s(M_{W_L}) = g^2/4\pi \approx 0.12$, equation (2) gives $M_{W_L}/M_{W_R} \approx 10^{10}$ which leads to proton lifetime* greater than $10^{36}$ years so that the present ongoing experiments are not sensitive enough to detect proton decay and Eq. (3) gives $\sin^2 \theta_w(M_{W_R}) \approx 0.27$ which falls way outside the experimental value [4].

$$\sin^2 \theta_w|_{\exp} = 0.215 \pm 0.012.$$ (4)

In deriving (2) and (3) the small contribution of $(M_{W_R}/M_{W_L}) (3 \leq M_{W_R}/M_{W_L} \leq 10)$ in the evolution of the gauge couplings is neglected. This leaves the 'multistage' descent of $G$ to the apparently observed $SU(2)_L \times U(1) \times SU(3)^f$. In this case $M_{W_R}$ is not constrained to lie between 250 GeV and a few TeV. The solution to the multistage hierarchy equations in terms of the weak angle $\sin^2 \theta_w$, strong coupling $\alpha_s$ and electromagnetic coupling $\alpha_m$ predicts [5] $M_{W_R} \geq 10^7$ GeV for the unifying mass-scale $M_w$ in the range $10^{13}$ GeV $\leq M_w \leq 10^{19}$ GeV. To illustrate this fact, first consider the fundamental unifying symmetry $G$ to be $SO(10)$ and the following hierarchy of spontaneous symmetry breaking [5]

$$M_{W_R} \approx 10^7 \text{ GeV} \quad \text{for the unifying mass-scale } M_w \text{ in the range } 10^{13} \text{ GeV} \leq M_w \leq 10^{19} \text{ GeV.}$$

* It is difficult to estimate the rate precisely since $M_w$ is greater than the Planck Mass ($= 10^{19}$ GeV). Gravitational effects are now important and are required to be taken into account. Precisely how this should be done remains the fundamental problem of the day.
\[
\text{SO(10)} \quad \downarrow M_w \\
\text{SU(2)}_L \times \text{SU(2)}_R \times \text{SU(4)}_c \downarrow \text{SU}(1) \downarrow M_r \\
\text{SU(2)}_L \times U(1)_R \times U(1) \times \text{SU(3)}_c \\
\text{SU(2)}_L \times U(1)_R \times \text{SU}(3)_c \\
U(1)^* \times \text{SU}(3)_c.
\]

The solutions in terms of \(\{\sin^2 \theta_W, \alpha_{em}, \alpha_s\}\) to the evolution of the gauge couplings of the various subgroups in (5) with boundaries in energy defined by the mass-scales \(\{M_\mu, M_{W_L}, M_{W_R}, M, M_r\}\) are

\[
\sin^2 \theta_W(M_{W_L}) = \frac{3}{8} + \frac{11 \alpha_{em}(M_{W_L})}{24\pi} \\
\cdot \left\{ \frac{2 \ln \frac{M_\mu}{M_{W_L}}}{2 \ln \frac{M_\mu}{M_{W_L}}} - 3 \ln \frac{M_{W_R}}{M_{W_L}} - \frac{4 \ln \frac{M_f}{M_{W_L}}}{M_{W_L}} \right\},
\]

(6)

\[
\frac{1}{8\pi} \left\{ \frac{11}{2 \ln \frac{M_\mu}{M_{W_L}}} + \ln \frac{M_{W_R}}{M_{W_L}} \right\},
\]

(7)

In deriving these equations, the definitions of the electric charge \(e\),

\[
e^{-2}(M_{W_L}) = g_L^{-2}(M_{W_L}) + g_R^{-2}(M_{W_L}) + (\frac{3}{8}) g_L^{-2}(M_{W_L}),
\]

(8)

and the weak angle \(\sin^2 \theta_W\),

\[
\sin^2 \theta_W(M_{W_L}) = e^{-2}(M_{W_L})/g_L^2(M_{W_L})
\]

are used.

For values of the unifying mass-scale \(M_s\) between \(10^{15}\) GeV (proton lifetime \(\approx 10^{34}\) yrs) and \(10^{19}\) GeV (highly stable proton with lifetime \(\sim 10^{46}\) yrs), \(\alpha_{em} \approx (128.5)^{-1}\), \(\alpha_s \approx 0.12\) and \(\sin^2 \theta_W \approx 0.23\), (6) and (7) predict \(M_R\) and \(M_f\) as below

\[
\frac{M_\mu}{M_{W_L}} = 10^{13} \quad 10^{15} \quad 10^{17}
\]

\[
\frac{M_{W_R}}{M_{W_L}} = 10^{13} \quad 10^{9.6} \quad 10^{5.6}
\]

\[
\frac{M_f}{M_{W_L}} = 10^{10} \quad 10^{14} \quad 10^{17}
\]

Thus with \(\{\sin^2 \theta_W, \alpha_{em}, \alpha_s\}\) deduced from phenomenology and extrapolated to energies equal to the weak interaction scale \(M_{W_L}\), the SO(10) unified theory does not allow the left-right symmetric strong and electroweak gauge symmetry \(G_{\text{SW}} = SU(2)_L \times SU(2)_R \times U(1) \times SU(3)_c\) to emerge at low energies \(M_{W_R} \approx 240\) GeV as hoped for initially. The lowest value of \(M_{W_R}\) at which \(G_{\text{SW}}\) manifests is of order \(10^5\) TeV and corresponds to the situation in which the proton is highly stable \(c_p \approx 10^{46}\) yrs. For lifetimes lower than \(10^{46}\) yrs \(M_{W_R}\) rapidly increases towards the \(10^{15}\) GeV limit. The gauge coupling ratio \(g_L^2/g_R^2\) of \(SU(2)_L\) and \(SU(2)_R\) is given by

\[
g_L^2(M_{W_L})/g_R^2(M_{W_L}) = 1 + \frac{11 \alpha_{em}(M_{W_L})}{3\pi} \ln \frac{M_{W_R}}{M_{W_L}}.
\]

(10)

It increases from its value of 1.5 at \(M_{W_R}/M_{W_L} \approx 10^{5.6}\) to 2.23 at \(M_{W_R}/M_{W_L} \approx 10^{13}\). The effect of such large deviations of \(g_L^2/g_R^2\) from unity have been discussed \([3]\) on neutrino and parity violating electron-nucleon neutral current interactions.

Next, consider the fundamental unifying symmetry \(G = E_6\) \([6]\). Although it admits more intermediate mass-scales, it shares the same fate as \(SO(10)\) as regards realising \(SU(2)_L \times SU(2)_R \times U(1) \times SU(3)_c\) as a low energy symmetry from \(E_6\). This can be seen by considering the following hierarchy of symmetry breaking in \(E_6\)

\[
E_6 \quad \downarrow M_e \\
SU(3)_L \times SU(3)_R \times SU(3)_c \downarrow \text{SU}(1) \downarrow M_3 \downarrow M_{W_L} \\
SU(2)_L \times \text{SU}(2)_R \times U(1)_L \times U(1)_R \times SU(3)_c \\
SU(2)_L \times U(1)_L \times \text{U}(1)_R \times SU(3)_c \\
SU(2)_L \times \text{U}(1)_R \times SU(3)_c \\
U(1)^* \times SU(3)_c.
\]

(11)

The intermediate mass-scales of Eq. (11) satisfy two constraint equations in terms of the empirical quantities \(\{\sin^2 \theta_W, \alpha_{em}, \alpha_s\}\). These are

\[
\sin^2 \theta_W(M_{W_L}) = \frac{11 \alpha_{em}(M_{W_L})}{6\pi} \ln \frac{M_{3L} + \alpha_{em}(M_{W_L})}{M_{W_L}} \alpha_s(M_{W_L}),
\]

(12)

\[
\ln \left( \frac{M_{3L} + M_{3R} + M_{W_R}}{M_{W_L}} \right) + \frac{3\pi}{11} \left\{ \frac{1}{\alpha_{em}(M_{W_L})} \right\} - \frac{8}{3\alpha_s(M_{W_L})},
\]

(13)

The ingredients of (12), (13) are the running coupling constants of the various sub-symmetries in (11).