Equal-Length Criterion for Transient Stability Detection

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Contents: The method suggested in this paper for detecting the transient stability of power systems can be considered as one of the most simple methods for this purpose. In this paper the power variations in the system are represented by circles and straight lines and the stability is detected by measuring lengths. This method is applied for different cases like the sudden load changes, switching operations and short circuits. This method gives an easier and simple alternative for detecting the transient stability compared by the equal-area criterion which requires the drawing a set of sine curves and the stability is detected by measuring areas bounded by these curves.

Ein Längen-Kriterium zur Ermittlung der dynamischen Stabilität


1. Introduction

One of the important problems in the analysis of power systems is the transient stability study of these systems. The study of the transient stability of power systems is not a simple matter whatever simple is this system. This is due to the nonlinearity of the differential equations describing the behaviour of the system during the transient periods.

There are numerous methods for solving these equations and studying the transient stability of the power systems, these methods differ in many ways. They differ on the field of application i.e. if it is required to study a simple system or an interconnected system, some methods are valued for the two cases, but the others are used only for the simple system. These methods differ also from the point of how much information its gives about the behaviour of the system during the transient period. Some methods give complete informations while the others indicate if the system is stable or not only. Some of these methods can be mentioned here for example, the equal area criterion [1], the phase plane method [2–5], the energy integral method [3, 4] the step-by-step method [1], and the Lyapinov functions method [5, 6].

The differential equations describing the behaviour of the power system during the transient period are the swing equations, the solution of these equations by the step-by-step method gives the swing curves which describe completely the behaviour of the system during the transient period and the stability of the system can be detected from these curves. This method is applicable for both simple and interconnected system. In majority of cases it is also required to know if the system is stable or not and no details is required about the behaviour of the system during the transient period, one of methods satisfying requirements is the equal area criterion.

The equal area criterion is applicable for simple system i.e. for a single machine or a group of machines connected in parallel and connected to and infinited bus directly or through a transmission system. It is also applicable for two groups of synchronous machines connected together through a transmission system, these groups can be reduced to one equivalent synchronous machine connected to an infinite bus through the transmission system. The study of the stability of this equivalent machine indicates the stability of the system.
In this paper we suggest another method which is similar in nature to the equal-area criterion, it only indicates if the system under study is stable or not without giving any details concerning the behaviour of the system during the transient period. It is also applicable for simple power systems. This method depends on measuring lengths instead of measuring areas which is the case of the equal area criterion. As measuring lengths is easier than measuring areas, the suggested method is expected to be simpler and less time-consuming than the equal-area criterion.

2. Theory

The usual assumptions for studying the transient stability of power systems are:

1) Representation of synchronous machines by constant voltage behind its transient reactance.
2) The flux linkage of the rotor of the synchronous machine is constant.
3) The mechanical input power to the machine is constant.
4) The angular momentum of the synchronous machine is constant during the transient period.
5) The resistances of the system are neglected.
6) All damping torques are neglected.

We are considering now the case of a single machine connected to an infinite bus through a transmission line. The swing equation of this system is

\[ M \frac{d^2 \delta}{dt^2} = P_a, \]

\[ = P_a - P_e, \]

\[ = P_a - P_m \sin \delta, \]

\[ = P_a - E_1 E_2 Y_{12} \sin \delta, \]

where

- \( M \) is the inertia constant of the machine
- \( P_a \) is the accelerating power
- \( \delta \) is the angular displacement, i.e. the angle between the voltage of the machine \( (E_1) \) and the infinite bus voltage \( (E_2) \)
- \( P_e \) is the input mechanical power
- \( E_1 \) the voltage behind the transient reactance of the machine
- \( E_2 \) is the infinite bus voltage
- \( Y_{12} \) is the total susceptance between \( E_1 \) and \( E_2 \)

Equation (1) can be written as follows:

\[ \frac{d}{dt} \left( \frac{d\delta}{dt} \right)^2 = \frac{2}{M} P_a \frac{d\delta}{dt}. \]

The angular velocity with which the machine swings apart is consequently

\[ \frac{d\delta}{dt} = \omega = \sqrt{\frac{2}{M} \int_{\delta}^{\delta_m} P_a \, d\delta}, \]

The machine will swing apart uptill this angular velocity becomes zero, if the system is stable, i.e.

\[ \int_{\delta}^{\delta_m} P_a \, d\delta = 0. \]

Eq. (2) as it stands, imposes no restriction on the input or the output of the machine. The former may be considered constant or variable, and the latter include damping and flux variations. The power differential \( P_a \) may be represented by functions with any arbitrary number of discontinuities. Hence the criteria inherently provides for inclusion of switching, fault clearing etc. As a matter of fact its validity definitely depends upon all operations causing discontinuities taking places before the maximum angle is reached.

Eq. (2) represents an area, in case of cartesian coordinates, between input and output curves, or lengths in case of polar coordinates, bounded by the initial and maximum angle. The equal area criterion, and also the suggested method, the equal length criterion, can be used for the detection of transient stability of the system when several types of disturbances might occur including:

1. Sudden load change
2. Switching operations
3. Short circuits.

Although the equal area criterion, and also the suggested method is not applicable to multi-machines system it helps to understand the factors affecting the transient stability of any system. The following section is a rapid review of the equal area criterion as a necessary introduction to the suggested method, the equal length criterion.

3. Equal Area Criterion

The three cases mentioned before, i.e. the sudden load change, switching operations, and short circuits which are the usual disturbances in any power system will be described here briefly as a necessary introduction to the suggested method, and also to become possible to make a comparison between the two methods.

3.1 Sudden Load Change

If a synchronous machine is connected to an infinite bus and the mechanical power is suddenly increased