An analysis on effect of transverse convex curvature on turbulent flow and heat transfer

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Abstract. An analysis based on a model of modified mixing length by Hornby, Mistry and Barrow [1] was made on the effect of transverse convex curvature in turbulent boundary layer for incompressible axial flows along circular cylinders. The deviation of various turbulent flow and heat transfer properties from those of flat plates is presented.

The agreement between the analyses and the experimental results for skin friction and heat transfer rate is good. The study demonstrated that, for a given condition, both the friction coefficient and Stanton number increase with decreasing value of the cylinder radius and that their values are always greater than those for the flow over a flat plate.

Eine Analyse der Wirkung konvexer Krümmung in Querrichtung auf turbulente Strömung und Wärmeübertretung


Nomenclature (see also figure 1)

$A^+ = \text{van Driest damping factor, } 26.0$
$c_x = \text{specific heat}$
$C_f = \text{friction coefficient}$
$F^* = \text{Eq. (25)}$
$h = \text{heat transfer coefficient}$
$k = \text{thermal conductivity}$
$K = \text{von Karman's constant}$
$Pr = \text{Prandtl number}$
$q = \text{heat flux}$
$Re = \text{Reynolds number}$
$r = \text{radius}$
$r_0 = \text{radius of test cylinders}$
$r_0^* = r_0 u_r/v$
$St = \text{Stanton number}$
$T = \text{temperature}$
$T^* = (T_w - T) c_p q u_r/q_w$
$u = \text{fluid velocity}$
$u_r = \sqrt{(u_w/q)}$

Subscripts

$fp = \text{flat plate}$
$h = \text{thermal}$
$M = \text{momentum}$
$ro = \text{cylinder}$
$t = \text{turbulent}$
$w = \text{wall}$
$x = \text{local}$
$\infty = \text{free stream}$

1 Introduction

It is generally accepted that the two-dimensional turbulence model can be directly used for prediction of fluid flow in axisymmetric turbulent boundary layers after Mangler transformation [2]. However, the transverse effect becomes significant when the radius of a body in a viscous flow is of the order of magnitude as the thickness of the boundary layer.

Transverse curvature (TVC) effects have been observed in two types of experiments:

i. The thick axisymmetric boundary layer developing on a long slender cylinder placed axially in a uniform stream [3], and

ii. the turbulent flow along the transversely convex surface of the core tube in annular space formed by two circular tubes [4, 5].
This implies that whereas the transverse concave curvature of the surface has little effect on the radial development of flow turbulence, that of the transverse convex curvature seems to be significant.

Experimental [6–8] and theoretical [9, 10] studies for the TVC effect on the fluid flow have been made by many. However, they are limited to velocity profiles and in the experimental studies, skin frictions is deduced using various indirect methods. Recently, there has been a few attempts [11, 12] to investigate turbulence structure in a thick axisymmetric boundary layer.

On the other hand, there seems to be hardly any study explicitly made on the effect transverse convex curvature on heat effect except that of an implicit analytical study made by Sparrow et al. [13]. In their study, the expression of Deissler for the eddy diffusivity was used, retaining the same constants used for the flow in pipes and over a flat plate, and therefore, the TVC effect was not explicitly recognized.

In the present study, the effect of transverse convex curvature on incompressible turbulent flows and convective heat transfer is analyzed based on a model of modified mixing length by Hornby, Mistry und Barrow [1]. The modified model is an extension of van Driest’s model for turbulent flow near the wall to cater for a wider range of flow geometries by including the influence of the whole boundary of the flow. For this reason, this particular model was used in the present analysis as it is the only turbulence model which recognizes the flow boundary shape.

The effect of transverse convex-curvature on incompressible turbulent flows and convective heat transfer is presented for the case of $\delta/r_0 \approx O(1)$ which has a wide range of application in engineering practice. The results from the analysis for cylindrical bodies of different diameter are compared with those of flows over a flat plate. The present analysis is also compared against our previous experimental results [14] as well as against the results of others [8, 13]. For heat transfer analysis, the numerical calculations are made for the Prandtl numbers of 0.7 and 7.

\section{Analysis}

For an axisymmetric turbulent boundary layer on a circular cylinder in a zero pressure gradient flow, the boundary layer equations for the mean flow are:

\begin{align}
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} &= \frac{1}{\rho} \frac{\partial (r v)}{\partial y} \\
\frac{\partial (r u)}{\partial x} + \frac{\partial (r v)}{\partial y} &= 0
\end{align}

where

$$\tau = \mu \frac{\partial u}{\partial y} - \rho u' v'$$

If the Reynolds stress is related to the mean velocity distribution through the concept of the mixing-length theory, Eq. (3) becomes, in dimensionless quantities, as:

$$\left( \frac{\partial u^+}{\partial y^+} \right)^2 + \left( \frac{\partial u^+}{\partial y^+} \right)^2 = \tau^+$$

which reduces to:

$$\frac{\partial u^+}{\partial y^+} = \frac{2 \tau^+}{1 + \sqrt{1 + 4 l^{+2} \tau^+}}$$

With the boundary conditions, $u^+ = 0$ at $y^+ = 0$,

$$u^+ = \int_0^y \frac{2 \tau^+ dv^+}{1 + \sqrt{1 + 4 l^{+2} \tau^+}}$$

To solve Eq. (6), the information on the mixing length, $l$, and shear stress distribution is needed.

\subsection{The mixing length model of Hornby, Mistry and Barrow [1]}

For the flow near the wall, various composite mixing-length and eddy-viscosity formulae have been suggested by a number of workers on the basis of empirical correlation of experimental observation. In the present study, the model proposed by Hornby, Mistry and Barrow [1] is used. Hornby, Mistry and Barrow extended the van Driest’s turbulence model which made use of Stokes Second Problem which is concerned with laminar flow near an oscillating plate. The solution of the momentum equation is known from the analogous heat conduction problem.

The van Driest’s model is basically for a flat plate and does not recognize the influence of the duct shape. To obtain a new model which takes cognisance of the duct shape, Hornby, Mistry and Barrow obtained the velocity distribution solving the laminar momentum equation with the whole duct oscillating. Once the fluid velocity was obtained, the damping factor for turbulent flow in ducts of any shape was acquired, applying van Driest’s analogy. The resulting damping factor is function of both wall distance and duct geometry.