On the Steady State Analysis of a Current Fed Induction Motor

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Contents: The performance equations of an induction motor operating on a current source inverter are derived using a stationary reference frame of two of the three phases. The equations are well suited for simulation of a current controlled inverter fed induction motor on a digital computer using well known numerical methods of integration. A method is presented for the steady state analysis via state transition signal flow graph. This method is found to offer several advantages over the methods available in the literature using state space techniques. The steady state solution of the equations can be used to show that the rotor flux varies sinusoidally and it induces a sinusoidal voltage in the stator phases accompanied with voltage spikes during commutation.

Über das stationäre Verhalten eines Asynchronmotors mit eingepäpptem Strom


Nomenclature

\[ \begin{align*}
    i_A, i_B, i_C & \quad \text{Stator phase currents} \\
    i_{a}, i_{b}, i_{c} & \quad \text{rotor phase currents} \\
    l_d & \quad \text{D. C. link current} \\
    l_q & \quad \text{mutual inductance between stator and rotor phases} (L_{\text{m}} = 3/2 l_q). \\
    L_{s}, L_{r} & \quad \text{leakage inductance of stator and rotor windings per phase} \\
    L_{s}, L_{r} & \quad \text{self inductances of stator and rotor windings per phase} \\
    R_s, R_r & \quad \text{stator and rotor resistances per phase} \\
    T_{\text{el}} & \quad \text{electromagnetic torque developed} \\
    u_A, u_B, u_C & \quad \text{stator phase voltages} \\
    u_{a}, u_{b}, u_{c} & \quad \text{rotor phase voltages} \\
    \Psi_A, \Psi_B, \Psi_C & \quad \text{stator flux linkages} \\
    \Phi_A, \Phi_B, \Phi_C & \quad \text{rotor flux linkages} \\
    \delta & \quad \text{electrical angle between the axes of the stator and rotor phases} \\
    \omega & \quad \dot{\delta} = \text{angular speed of the rotor} \\
    \end{align*} \]

1 Introduction

With the advent of thyristors and thyristor inverters it has become possible to control the speed of an induction motor over a wide range very smoothly. The drive utilizing inverter-fed induction motor has thus become very popular. It is almost a rival to the drive employing a D.C. shunt motor. The inverter-fed motors operate on nonsinusoidal excitation and are subject to increased losses and harmonic torque oscillations.

A study of the transient as well as the steady state behaviour of an inverter-fed induction motor is very useful as it provides the necessary information regarding the voltages and currents of the inverter which solely govern the efficient and optimum selection of the thyristors and commutation circuit elements of the inverter.

The performance characteristics of induction motors operating on periodic nonsinusoidal current excitation were analysed using a variety of methods [1—6]. These existing methods of approach assume the input current to vary in steps treating the waveform to be piecewise constant over a certain period. Naunin [1, 2] and Klautschek [3] using the concept of space vector gave a closed-form solution to the differential equations of a current-fed induction motor. Lipo and Cornell [4] based their approach on state variables to analyse a current controlled induction motor. The solution to the equations is given in terms of state transition matrix and convolution integral. Fourier analysis [5] of nonsinusoidal functions was also used to analyse the inverter-fed induction motor. Sattler and Ulrich [6] used analog computer simulation for obtaining the performance of an induction
motor fed from a voltage source inverter. The simulation was carried out using 3 phase equations without resorting to two axis transformations.

The aim of the present paper is three fold:

i) It is proposed to derive the equations describing the performance of a current controlled induction motor in 3 phase variables. These equations are very useful for the digital simulation of the inverter-machine combination and also when a machine with a number of phases exceeding three is encountered.

ii) An alternate method for obtaining the state transition matrix based on the state transition signal flow graph is developed. A closed form solution is made available for the performance of a current controlled induction motor under steady state. As will be explained later this method has several advantages over already available methods in the literature.

iii) The assumption that the input current of a current-fed induction motor has a stepped waveform is not strictly correct. During commutation of the current from one phase to the other the machine inductance and commutating capacitance make it impossible to have the instantaneous current transfer. Thus the input current waveform tends to be trapezoidal. Under these circumstances the state variable approach due to Lipo and Cornell creates a formidable mathematical problem of requiring evaluation of convolution integrals during commutation intervals. On the other hand the present method can very effectively handle this type of input and the solution can be obtained in a closed-form. In the present paper the input current waveform is assumed to be trapezoidal.

2 Commutation Process in the Current Source Inverter and Stator Current Waveforms of the Induction Motor

The current source inverter feeding a three phase induction motor is shown in Fig. 1. The inverter is of the autosequential commutated type according to Ward [?] and is becoming very popular with the advent of thyristors having large peak inverse voltages.

The following assumptions are made in the discussion of commutation process in the inverter:

a) The diodes and thyristors used in the inverter are ideal switches.

b) The D.C. link current fed to the inverter is constant and contains no ripples.

c) Commutation overlaps do not occur.

It is assumed that the thyristors T1 and T6 are conducting to begin with and the thyristor T3 is fired to commutate T1 at the instant \( t = t_a \). The commutation of the current is completed in three stages. The variation of current during commutation is shown in Fig. 2.

In the first stage the current commutates from T1 to T3 in a short interval of time \( t_b \) as the voltages across capacitors C4 and C5 act as reverse voltages to the thyristor T1. In general \( t_b \) is very small compared to the total commutation period and it can be neglected. In other words the commutation of current from thyristor T1 to the thyristor T3 is almost instantaneous.

At any instant \( t > t_b \) the link current through the thyristor T3 flows through the commutating capacitors (C1 parallel to the series combination of C3 and C5) as diode D3 is reverse biased. The current through the capacitors modifies the voltage across the capacitors. Capacitor C1 gets discharged to zero at the instant \( t = t_c \). The period \( t_{de} \) is generally

![Fig. 1. Current source inverter feeding a 3-phase induction motor](image1.png)

![Fig. 2. Variation of line currents of a 3-phase induction motor fed from a current source inverter during commutation](image2.png)