Hadron correlators and the structure of the quark propagator*

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Abstract. The structure of the quark propagator of QCD in a confining background is not known. We make an ansatz for it, as hinted by a particular mechanism for confinement, and analyze its implications in the meson and baryon correlators. We connect the various terms in the Källen-Lehmann representation of the quark propagator with appropriate combinations of hadron correlators, which may ultimately be calculated in lattice QCD. Furthermore, using the positivity of the path integral measure for vector like theories, we reanalyze some mass inequalities in our formalism. A curiosity of the analysis is that, the exotic components of the propagator (axial and tensor), produce terms in the hadron correlators which, if not vanishing in the gauge field integration, lead to violations of fundamental symmetries. The non observation of these violations implies restrictions in the space-time structure of the contributing gauge field configurations. In this way, lattice QCD can help us analyze the microscopic structure of the mechanisms for confinement.

1 Introduction

The structure of the quark propagator of QCD in a confining background has been a matter of debate over the years [1]. The various mechanisms for confinement hint different types of vacua and therefore different quark propagators [1, 2]. In particular the electric vortex mechanism (see Appendix A) provides us with a quark propagator with the following Källen-Lehmann representation,

\[ S(x, y) = s(x, y) + v_\mu(x, y) \gamma_\mu + a_\mu(x, y) \gamma_\mu \gamma_5 + t_{\mu\nu}(x, y) \sigma_{\mu\nu}. \]  

We shall take this as an ansatz for the structure of the quark propagator in a background to be used in the formalism of the so called QCD inequality approach.

It was realized long after QCD was formulated that one could derive some exact inequalities between hadron masses [3] and other observables [4]. The key element in deriving them is that the Euclidean fermion determinant in vector like gauge theories (such as QCD) is positive definite and so the measure

\[ d\mu = Z^{-1} DA_\mu(x) \det(i\partial + M) \exp \left( -\frac{1}{2g^2} \int d^4x Tr F_{\mu\nu}^2 \right) \]  

for the \( A_\mu \) integration obtained after integrating out the fermions is positive definite for \( \Theta = 0 \). Note that \( \partial = \gamma^\mu D_\mu \), \( D_\mu \) being the covariant derivative. Inequalities that hold pointwise continue to hold after integrating with respect to a positive measure. Thus any inequality among matrix elements that holds after performing the Fermi integral in a fixed background gauge field holds in the exact theory. The continuous formulation requires from an appropriate regularization scheme [5]. The great advantage of this procedure is that one sums over positive contributions weighted by a positive measure and therefore possible cancellations between different gauge configurations are not worrysome.

The aim of this paper is to analyze various consequences of (1) within the inequality approach for QCD*. Our interest is twofold. On the one hand we shall discuss properties of QCD, i.e., chiral symmetry realization, mass relations, ..., as if the above ansatz were the outcome of the true calculation. On the other we shall relate the terms in the ansatz to hadron correlators, which can ultimately be calculated in lattice QCD. Finally we shall discuss observable consequences of the exotic terms in the Källen-Lehmann representation of the quark propagator, which imply, to avoid violation of fundamental symmetries, a strong restriction of the space-time structure of the contributing gauge field configurations.

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2 The structure of the mesonic correlators

Mass inequalities have been obtained among the mesons and comparing baryons with mesons [3]. The important property in these calculations has been

\[ S^+(x, y) = \gamma_5 S(y, x) \gamma_5, \]  

(3)

where \( S(x, y) \) is the quark propagator in a background. Our aim is to discuss mass relations also among baryons. In this case due to their current quark constituency the previous property is of no use. Some fine details of the structure of the quark propagator and of the baryon currents will be necessary to be able to address the issue. We proceed thus with the same technique in both cases, only that in the meson case we will use of this simplification.

The meson correlators in terms of meson fields are given by*

\[ \langle \sigma(x) \sigma(y) \rangle = - \int d\mu(\gamma_5 S^+(x, y) \gamma_5 S(x, y)), \]  

(4)

\[ \langle \pi(x) \pi(y) \rangle = \int d\mu(\gamma_5 S^+(x, y) S(x, y)), \]  

(5)

\[ \langle \rho_\mu(x) \rho_\nu(y) \rangle = \int d\mu(\gamma_5 S^+(x, y) \gamma_5 \sigma_{\mu\nu} S(x, y) \gamma_5), \]  

(6)

\[ \langle \pi(x) \pi(y) \rangle = - \int d\mu(\gamma_5 S^+(x, y) \gamma_5 S(x, y) \gamma_5), \]  

(7)

\[ \langle \tau_\mu(x) \tau_\nu(y) \rangle = - \int d\mu(\gamma_5 S^+(x, y) \gamma_5 \sigma_{\mu\nu} S(x, y) \gamma_5). \]  

(8)

According to our previous discussion one should investigate the properties of these correlators for the quark propagator in the presence of a background field. The substitution of (1) and (3) into (4) through (8) provides us with the structure of the mesonic correlators. With the help of Mathematica and HIP [6] the calculation is straightforward (see Appendix B). We discuss here some of the properties of the arising structures.

It was noticed some time ago [7] that the difference between the sigma and pion correlators

\[ \langle \pi \pi \rangle - \langle \sigma \sigma \rangle = 2 \int d\mu(|s|^2 + 2|t|^2), \]  

(9)

could be non vanishing if anomalous structures were present in the quark propagator. In such a case chiral symmetry would be broken by the mechanism leading to these structures. In our case the electric vortex contributes both to \( s \) and \( t_{\mu\nu} \) leading to the spontaneous breaking of chiral symmetry. However this contributions are proportional to the fermion mass, thus our mechanism could never explain the spontaneous breaking of chiral symmetry in a massless theory. However this is not an inconvinence, since as can be seen in [9], the mass plays also a crucial role in more fundamental approaches. In the present model the restoration of chiral symmetry and deconfinement would occur at the same scale. However this statement has to be taken with precaution due to our simplified scenario. It could happen that other mechanisms, like for example instanton effects, could modify the conclusions [10].

* Since global numerical factors are of no relevance for our discussion, we later on normalize the measure \( d\mu \) so that the coefficient of the scalar term is unity for the trace correlator.