This paper presents a “branch and bound” method for solving mixed integer linear programming problems. After briefly discussing the bases of the method, new concepts called pseudo-costs and estimations are introduced. Then, the heuristic rules for generating the tree, which are the main features of the method, are presented. Numerous parameters allow the user for adjusting the search strategy to a given problem.

This method has been implemented in the IBM Extended Mathematical Programming System in order to solve large mixed integer L.P. problems. Numerical results making comparisons between different choices of rules are provided and discussed.

1. Introduction

This paper presents a “branch and bound” method for solving mixed integer linear programming problems. After briefly discussing the bases of the method, new concepts called pseudo-costs and estimations are introduced. Then, the heuristic rules for generating the tree, which are the main features of the method, are presented. Numerous parameters allow the user to adjust the search strategy for a given problem.

This method has been implemented in the IBM Extended Mathematical Programming System in order to solve large mixed integer L.P. problems. An early version of this method, not including all the features described here, has been adapted in a code presently available in all IBM World Trade Data Centers. Numerical results making comparisons between different choices of rules are provided and discussed.

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2. "Branch and bound" method

2.1. Problem Statement

Any mixed integer linear program can be written in the following way:

Let \( X \) and \( Y \) denote two column vectors the components of which are \( x_i, i = 1 \) to \( p \) and \( y_j, j = 1 \) to \( q \) respectively. The two rectangular matrices \( A \) and \( B \) are of the order \((m, p)\) and \((m, q)\) respectively. Then the problem is:

**Problem M**

\[
\min F = A^T X + B^T Y \tag{2.1}
\]

under the constraints

\[
AX + BY = D \tag{2.2}
\]

\[
\alpha_j \leq y_j \leq \beta_j \tag{2.3}
\]

\[
y_j \text{ integer } \quad j = 1 \text{ to } q. \tag{2.4}
\]

\[
0 \leq x_i \quad i = 1 \text{ to } p. \tag{2.5}
\]

The \( x_i \) are the continuous variables, whereas the \( y_j \) are the integer variables. The bounds over the \( y_j \) must be finite, but can be either positive or negative. The problem, obtained when removing the integrality condition (2.4), is called the continuous problem \( C \).

An integer solution of \( M \) or \( C \) is a set of values for the \( x_i \) and the \( y_j \) satisfying (2.2) to (2.5). An optimal integer solution is an integer solution minimizing \( F \) (in this paper, minimization will always be assumed).

2.2. Stages of the Method

The method involves two stages:

- First an optimal continuous solution of \( C \) is searched for by means of the usual linear programming methods. If this solution is integer, problem \( M \) is solved. Assume this is not the case.
- Then, an ordered sequence of continuous linear programming