AN ALGORITHM FOR COMPUTING EQUILIBRIA IN A LINEAR MONETARY ECONOMY

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An algorithm is presented for computing equilibria in a linear monetary economy, that is, an exchange economy in which all individuals have linear utility functions and in which goods are bought and sold only in exchange for money. The algorithm computes the equilibrium prices by solving a finite sequence of linear programming problems.

Key words: Equilibrium, Linear Monetary Economy, Linear Programming.

1. Equilibria in a linear monetary economy

In this paper, we develop a method to compute price equilibria for linear monetary economies. By linear monetary economy, we mean a pure exchange economy in which goods can be bought and sold only in exchange for money, and in which all individuals have linear utility functions for goods and money. An equilibrium of such an economy should be interpreted as a short-run equilibrium or a one-period temporary equilibrium in a dynamic economy, so that the individuals' utility for money is derived from their expected use of money in future periods. Temporary monetary equilibria of this form, with a "cash in advance" constraint on individuals' demand, have been proposed by Clower [1]; see also [3]. Our algorithm may be used in simulating such models.

Many algorithms have been suggested for computing economic equilibria, [4]. Eaves [2] has shown that Lemke's algorithm can be used to solve general linear exchange economies in finitely many steps. We shall show here that linear monetary economies have enough special structure so that computing their equilibria can be reduced to solving a finite sequence of linear programming problems. In fact, when one has a good first estimate of what the equilibrium prices might be (as would be the case in simulations of a dynamic model, where last period's prices could be used), our algorithm could require only one linear program to converge.

Let I denote the number of individuals in the economy, with i denoting a typical individual, so that i always ranges over $i = 1, \ldots, I$. Let J denote the number of goods and other non-money assets, with j denoting a typical non-money asset, so that j always ranges over $j = 1, \ldots, J$. We may think of money as asset #0.
Let $X_{ij}$ denote the quantity of asset $j$ which individual $i$ has available to sell this period, and let $X_{io}$ denote the quantity of money which $i$ has available to spend this period. We assume that each individual $i$ has a linear utility function for assets, and so let $U_{ij}$ denote $i$'s marginal utility for asset $j$. We shall assume that all $U_{ij} > 0$, which will guarantee that any equilibrium will have all positive prices. Without loss of generality (rescaling the individual's utility functions if necessary), we assume that each individual has unit marginal utility for money, so that $U_{00} = 1$.

Our problem is to compute prices, supply and demand in a market equilibrium. We shall let $p_j$ denote the price of asset $j$. Our supply and demand variables will be expressed in terms of the market value of the quantities supplied and demanded (evaluated at the $p_j$ prices), rather than in physical units. That is, $s_{ij}$ will denote the value of the quantity of asset $j$ which individual $i$ supplies to the market, and $d_{ij}$ will denote the value of the quantity of $j$ which $i$ demands from the market. Thus, the actual quantities of $j$ supplied and demanded by $i$ would be $s_{ij}/p_j$ and $d_{ij}/p_j$ respectively.

Given the market prices $p_j$, each individual $i$ wants to choose his $s_{ij}$ and $d_{ij}$ quantities so as to maximize

$$\sum_{j=1}^{J} \left( \frac{U_{ij}}{p_j} - 1 \right) (d_{ij} - s_{ij}),$$

subject to

$$s_{ij}/p_j \leq X_{ij} \quad (\forall j = 1, \ldots, J),$$

$$\sum_{j=1}^{J} d_{ij} \leq X_{io},$$

$$s_{ij} \geq 0 \quad \text{and} \quad d_{ij} \geq 0 \quad (\forall j = 1, \ldots, J).$$

To see where the coefficients in (1) come from, observe that every dollar (or unit of money) spent to buy $j$ brings in $1/p_j$ units of $j$, each of which contributes $U_{ij}$ units of utility, while the loss of the dollar spent reduces $i$'s utility by 1 (since $U_{00} = 1$). Thus $(U_{ij}/p_j - 1)$ is the coefficient of $d_{ij}$ in $i$'s utility formula. Similarly, every dollar's worth of $j$ sold contributes one unit of utility for the money brought in, but costs $U_{ij}/p_j$ units of utility for the $1/p_j$ units of $j$ sent out. So $(1 - U_{ij}/p_j)$ is the coefficient of $s_{ij}$ in (1). Constraint (2) asserts that the quantity of $j$ sold by $i$ cannot exceed $i$'s available endowment of $j$. Constraint (3) asserts that $i$'s total spending cannot exceed his available money balances. Constraint (4) gives the obvious non-negativity constraints. (This interpretation of (1)–(4) is in the spirit of [1], but differs slightly from [3]. It is easy to see that the individual's decision problems in [3] are mathematically equivalent to (1)–(4), with a simple translation of notation.)

The 'marginal value' $U_{ij}$ in this paper corresponds to the ratio $v_{ij}(t)/v_{io}(t)$ in [3], and the 'endowments' $X_{ij}$ and $X_{io}$ in this paper correspond to the maximum transaction rates $n_{ij}x_{ij}(t)$ and $n_{io}x_{io}(t)$ in [3]. What we have labelled $s_{ij}$ and $d_{ij}$ in this paper would correspond to $p_j s_{ij}(t)$ and $p_j d_{ij}(t)$ in the notation of [3], where the supply and demand variables are measured in physical units.