A PARTITIONING ALGORITHM FOR
THE MULTICOMMODITY NETWORK FLOW PROBLEM

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A partitioning algorithm for solving the general minimum cost multicommodity flow problem for directed graphs is presented in the framework of a network flow method and the dual simplex method. A working basis which is considerably smaller than the number of capacitated arcs in the given network is employed and a set of simple secondary constraints is periodically examined. Some computational aspects and preliminary experimental results are discussed.

1. Introduction

The multicommodity network flow problem has existed in the sphere of operations research since the 1950’s. It arises from many different applications, in particular from those dealing with transportation. Urban traffic assignment problems [2] and school desegregation problems [3] have been so formulated. An application to tanker scheduling is reported in [1]. The work presented here was stimulated primarily by the problems associated with routing freight cars over a railroad, where different car classes must be distinguished and where length or weight restrictions are imposed on the trains [28].

Since the initial suggestion by Fulkerson [7], many techniques for solving this problem have appeared in the literature. Most of the approaches have been derived from this early suggestion and from its more general result, the decomposition principle of Dantzig and Wolfe.
[6], including a number of rediscoveries of the equivalence between arc-chain formulations and the application of this decomposition technique. Some approaches which have utilized decomposition appear in [26, 29], although no computational work has been reported. However, the slow convergence characteristics of the decomposition algorithm for related problems, e.g. [12], indicate that other approaches may be more effective. Furthermore, it is well known that problems with naturally integral basic solutions may fail to retain this property at the conclusion of the decomposition algorithm.

Another approach was initiated by [18], who used a specialization of the primal-dual simplex method. Other primal simplex specializations have also been developed [23, 15]. A “resource directed” method appears in [24]. In a related area, communication network problems have been considered by many people and Hu [16] has solved the two commodity case. The special case when all commodities come from a single given source has also been investigated in [30].

In this paper we consider the general multicommodity flow problem with “weighted” coupling constraints on arc flows. The subproblems are assumed to be general networks with exogenous flows at each node, which seem to be more common in practice as illustrated by the railroad application. Such networks specialize to both maximum flow and single source–single sink networks merely by altering the cost function and the right hand side. The reverse is not true unless individual arc bounds are included, which complicate the existing algorithms.

The proposed algorithm generalizes the dual simplex method to take advantage of the well known observation that, in practice, only a small subset of the mutual arc capacity constraints are actually binding (the arcs are “saturated”) in an optimal solution. Accordingly, a large number of these constraints, initially (but quite arbitrarily) assumed to be inactive are set aside as “secondary” constraints. The remaining mutual arc capacity restrictions, along with the network subproblem equations (which form a weakly coupled block diagonal system) are denoted as the current constraints.

Dual simplex steps in the current problem are performed by employing a generalization of Primal Partition Programming [22]: The current and secondary arc capacity constraints are reviewed periodically to decide, according to various computational strategies, whether a new reclassification is necessary. All computations are performed by employing a minimum cost network flow algorithm, by manipulating