FAST COMPUTATION OF RAY SYNTHETIC SEISMOGRAMS IN VERTICALLY INHOMOGENEOUS MEDIA

Vlastislav Červený, Jaromír Janský

Institute of Geophysics, Charles University, Prague*)

Summary: A procedure of fast computation of body-wave ray synthetic seismograms in vertically inhomogeneous media is suggested. The procedure uses a special approximation of the velocity-depth distribution which guarantees continuity of the first and second derivatives of velocity and does not generate false low-velocity layers (oscillations in the velocity-depth function). The ZESY82 program package, which is based on the suggested procedure, is described. The point source with an arbitrary radiation pattern may be situated at any points of the model, the receivers are situated regularly or irregularly along any profile on the Earth's surface, containing the epicentre. Numerical examples of the synthetic record sections for a model of the Earth's crust and the uppermost mantle are given.

1. INTRODUCTION

To solve direct and inverse seismic problems in vertically inhomogeneous media, it is useful to have a stable and fast algorithm, even though approximate, for the evaluation of ray synthetic seismograms. The stability and efficiency of computation of synthetic seismograms, however, depend strongly on the method of approximation of the velocity-depth distribution. The standard methods of approximation do not usually guarantee the continuity of the first and second derivatives of the velocity. These features cause difficulties in numerical evaluation of ray integrals and produce false anomalies in the amplitude-distance curves. The same complications occur in the computation of body-wave ray synthetic seismograms.

A new approximation of the velocity-depth distribution would be very useful that would overcome these difficulties and give more reasonable amplitude-distance curves and, consequently, more reasonable synthetic body-wave ray seismograms. A suitable approximation was suggested recently in [1]. It reads

\[ z = a_j + b_j v^{-2} + c_j v^{-4} + d_j v^{-6} \quad \text{for} \quad z_j \leq z \leq z_{j+1}, \]

where \( z \) is the depth and \( v = v(z) \) denotes the velocity, \( j = 1, 2, \ldots, n \). In those parts of the medium where the velocity varies smoothly and monotonously with depth, the coefficients \( a_j, b_j, \ldots \)

*) Address: Ke Karlovu 3, 121 16 Praha 2.
c_j, d_j may be calculated by the smoothed spline approximation [2, 3]. The approximation (1) then guarantees the continuity of the velocity-depth distribution with its first and second derivatives (even across the grid points) and the stability of the amplitude distance curve.

The procedure of computing synthetic seismograms described here is based on ray methods. The wave field, generated by a point source situated inside the model or on its surface, is computed as a superposition of elementary waves (reflected, refracted, converted, etc.) corresponding to the zero-order approximation of the ray method.

Let us now consider one elementary wave. If the ray parameter p is given, the computation of the ray integrals and of their derivatives with respect to the ray parameter (required to evaluate geometrical spreading) is very fast for the approximation described above, as the integrals can be simply evaluated analytically. These analytical expressions do not contain any transcendental functions, only square roots and some simple polynomials.

In the computation of synthetic seismograms, however, the ray parameters are not known in advance, only the positions of the source and the receivers are specified. In this way, it is first necessary to determine the ray parameters of all rays of the elementary wave under consideration connecting the source and the receivers (two-point ray tracing). We must remember that any elementary refracted wave may arrive several times at the same receiver (loops in arrival times) and that some receivers may be situated in a shadow zone, where no ray of the elementary wave arrives.

As soon as the ray parameters are found, it is simple to determine the arrival times and ray amplitudes of a given elementary wave at the receivers.

The contents of this paper are as follows. In Sec. 2, we describe the model, the source and the receivers. In Sec. 3, the algorithm of the determination of ray parameters is discussed. The equations for evaluating travel-times and complex-valued amplitudes at the receivers (assuming the ray parameters are known) are summarized in Sec. 4. In Sec. 5, the numerical codes of elementary waves and their generation by the computer are shortly described. A short description of the ZESY82 program package, which is based on the suggested procedure, is given in Sec. 6 and some numerical examples can be found in Sec. 7. Finally, possible modifications of program ZESY82 are discussed in Sec. 8.

Let us note that the ZESY82 program package broadly uses several computer routines and algorithms of another program package, SEIS81, see [7]. The SEIS81 program package is designed for the computation of body-wave synthetic seismograms in 2-D laterally varying structures. Even though the SEIS81 program package computes the synthetic seismograms for more general models, it was possible to exploit several of its procedures and routines even in ZESY82, with slight modification only. This applies mainly to the routine for determining ray parameters. In SEIS81, the rays are determined by ray tracing, whereas ray tracing is not necessary in ZESY82; the ray integrals yield the epicentral distance directly (for a specified ray parameter) without evaluating the rays. In spite of this, the routine for determining ray parameters, used in SEIS81, can be used here too. It was also possible to use the routine for generating numerical codes of elementary waves (see Sec. 5), the routine for evaluating reflection/transmission coefficients, the SYNTPL program for evaluating synthetic seismograms for a realistic source-time function from synthetic impulse seismograms, the SEISPLOT program for plotting synthetic seismograms, etc.

2. MODEL, SOURCE, RECEIVERS

The model is specified in the Cartesian coordinate system x, z, where z is the depth, increasing downwards, x is the horizontal coordinate, increasing from the left to